Pre Calculus 12
Conics - Ellipse
Draw this 3 ways with a rope.
Horizontal, vertical and circle
Horizontal Ellipse

At (0, 0): \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

General: \[ \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \]
\[ a^2 - b^2 = c^2 \]

Center: \((h, k)\)  Foci: \((h \pm c, k)\)
Vertices: \((h \pm a, k)\)  Co-Vertices: \((h, k \pm b)\)
Vertical Ellipse

At (0, 0):
\[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \]

General:
\[ \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \]
\[ a^2 - b^2 = c^2 \]

Center: \( (h, k) \)  
Foci: \( (h, k \pm c) \)  
Vertices: \( (h, k \pm a) \)  
Co-Vertices: \( (h \pm b, k) \)
Sample Question

Find the center, vertices, foci, then draw the following ellipse:

\[
\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1
\]

First since 6 > 4 we know that the major axis is horizontal.

Now we use the formula sheet to find:
Center: \((h,k) = (0,0)\)
Vertices: \((h±a, k) = (±6,0)\)
Before we find the foci, first we need to find \(c\).

\[a^2 = b^2 + c^2\]
\[5^2 = 4^2 + c^2\]

so \(c = 3\).

Foci: \((h±c, k) = (±3, 0)\)
A couple more

\[
\frac{(x - 3)^2}{4} + \frac{(y + 2)^2}{9} = 1
\]

\[
4x^2 + 8x + 9y^2 - 54y + 49 = 0
\]

Just in case.

\[
x^2 + y^2 = 25
\]