CH5 - Math 12 PC

5.1 Graphing Exponential Functions

$y = a^x$ or $\frac{1}{2}^x$ * $y = 1$ when $x = 0$

So y intercept = 1

* Also - y can never = 0
so there is horizontal asymptote at $y = 0$

5.2 Analyzing Exponential Functions

* recall $y - k = af(b(x-h))$ → different variable but same position & meaning!

$y = a^x$ (base equation)

translations → $y - k = ca^{d(x-h)}$

horizontal (right −, left +)

horizontal stretch or compression

vertical (up −, down +)

vertical stretch or compression

general translation formula $\left(\frac{x}{d} + h, cy + k\right)$

NOTE * when choosing points
- include x & y intercepts
& choose both positive & negative values.

Also horizontal asymptote will have to be moved if there is a K value.

$\boxed{5.3}$ Solving Exponential Equations

To solve exponential equations make both sides of the same base.

ex 1 $2^x = \frac{1}{128}$ ← write in a power of 2 $\quad 128 = 2^7$

change 1 base $\quad 2^x = \frac{1}{2^7} \Rightarrow 2^x = 2^{-7} \quad \therefore x = -7$

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ex 2 $4^x = 8^{x-1}$ $\quad$ * 8 can't be written in a power of 4 <u>but</u> both 4 & 8 can be written in a power of 2

change both bases

$2^{2x} = 2^{3(x-1)}$

$2^{2x} = 2^{3x-3}$ $\quad$ * once the bases are the same use algebra using only powers

$$2x = 3x - 3$$
$$-3x \quad -3x$$
$$-x = -3$$
$$\therefore x = 3$$

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ex 3 $2^x = 4\sqrt{2} \longrightarrow \sqrt{2} = 2^{\frac{1}{2}}$

$4 = 2^2$

radicals + fraction exponents

$2^x = 2^2 \cdot 2^{\frac{1}{2}}$ ← add exponents when bases are = $\quad 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

$2^x = 2^{\frac{5}{2}} \quad \therefore x = \frac{5}{2}$ or $2\frac{1}{2}$

$\boxed{5.3}$ Compound Interest

$$\underbrace{A}_{\text{original amount + interest}} = \underbrace{A_0}_{\text{original \$ put in}}\left(1+\underbrace{\frac{i}{n}}_{\text{interest rate (decimal form)}}\right)^{nt}$$

t = time in years

n = # times compounded (annual = 1, bi/semi = 2, quarterly = 4, monthly = 12, daily = 365, weekly = 52)

A_0 (Principal) = \$1000 ; $i = 6\%$; $t = 5$ years ; n = monthly.

* use order of operations!

$A_0 = 1000$
$i = 0.06$
$t = 5$
$n = 12$

$$A = 1000\left(1+\frac{0.06}{12}\right)^{12 \times 5 = 60}$$
$$= 1000(1.005)^{60}$$
$$= 1000(1.3488) \Rightarrow A = 13,48.85$$

$\boxed{\text{Growth Factor}} \Rightarrow y = ak^n$

k → whole number = growth

k → fraction/decimal = decay (<1 but >0)

* When writing exponential equations remember if it will be increasing the K will be > 1

* if an event will decrease by 5% this means 95% will be left so K = 0.95 not 0.05

$\boxed{5.4}$ Logarithms & the Logarithmic Function

Logarithms are the INVERSE of an exponential function.

The Inverse of $y = 10^x$ is $y = \log_{10} x$

Note the pattern in the following

$\log_{10} 1 = 0 \quad \cdots\!> \quad 10^0 = 1$

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

base — answer — exponent

Knowing this → $\log_2 8 = 3$

$\therefore 2^3 = 8$

$$\log_b c = a \longrightarrow c = b^a$$

* Now go backwards →

$2^5 = 32 \Rightarrow \log_2 32 = 5$

$\log_b b^n = n$ ← equal

same

$\log_2 2^4 = 4$

5.4

ex 1 $\log_3 729$ ← make into an power with base 3

$729 = 3^6$

$\therefore \log_3 3^6 = 6$

ex 2 $\log_2(\sqrt[3]{4})$ make into base 2

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 2^{2(\frac{1}{3})} = 2^{\frac{2}{3}}$

$\therefore \log_2 2^{2/3} = \frac{2}{3}$

To graph a logarithmic function remember that it the inverse of the exponential function

So $y = \log_3 x$ (switch x & y values for $y = 3^x$)

$y = 3^x$

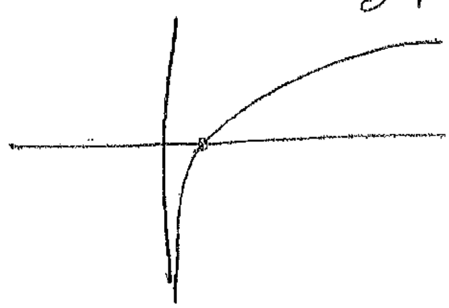
x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

← Switch

$y = \log_3 x$

x	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

* domain is always $x > 0$



$\boxed{5.5}$ The Laws of Logarithms

$b > 0$
$b \neq 1$

Product Law: $\log_b xy = \log_b x + \log_b y$

Quotient Law: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Power Law: $\log_b x^k = k\log_b x$

and $k\log_b x = \log_b x^k$

* to use these laws $\underline{\underline{b}}$ has to be the same!

ex 1

power law

$2\log x - \log y$

quotient law

$\log x^2 - \log y \Rightarrow \log\left(\frac{x^2}{y}\right)$

ex 2

power law

$\frac{1}{2}\log x - 3\log y + 2\log z$

$\log x^{\frac{1}{2}} - \log y^3 + \log z^2$

quotient law

$\log\left(\frac{x^{\frac{1}{2}}}{y^3}\right) + \log z^2$

product law

$= \log\left(\frac{x^{\frac{1}{2}} z^2}{y^3}\right)$

NOTE

* If given a whole number determine log with same base that equals whole #

$2 + \log_4 3$

$\log_4 \square = 2 \Rightarrow 4^2 = 16$

so $\therefore$ $2 = \log_4 16$

$\log_4 16 + \log_4 3 = \log_4 (16 \cdot 3)$

$\boxed{5.6}$ Analyzing Logarithmic Functions

$$y-k = c\log_a d(x-h)$$

$|c|$ - vertical stretch — $c<0$ reflect x-axis

$\frac{1}{|d|}$ - horizontal stretch — $d<0$ reflect y axis

k - vertical translation

h - horizontal translation

$$\left(\frac{x}{d}+h,\ cy+k\right)$$

$$\boxed{y=\log_3(2x+6) \Rightarrow y=\log_3 2(x+3)}$$

* remember to create table of values for exponential function then flip or create a table of values for

$3^x = y$

-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$3^y = x$

$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

same $\Longrightarrow$

$y=\log_3 x$

$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

translate

x	y
* $\frac{x}{d}+h$	$cy+k$
* $\frac{x}{2}-3$	y
$-\frac{53}{18}$	-2
$-\frac{17}{6}$	-1
-2.5	0
-1.5	1

$\boxed{5.7}$ Solving Logarithmic and Exponential Equations

* You can add and subtract logs just like x and y.

$x + x = 2x \Rightarrow \log_2 x + \log_2 x = 2\log_2 x$

Use this information to solve

$\log_2 x + \log_2 x = 2 \Rightarrow 2\log_2 x = 2$

$\div 2 \quad \div 2$

$\Rightarrow \log_2 x = 1$ * put in exponential form

$x = 2^1$

$\therefore x = 2$

*Change base

$\log_a b = \dfrac{\log b}{\log a}$

ex 1 $\quad 5 = \log_2 x + \log_2 2x$ *use product law

$\Rightarrow 5 = \log_2(x \cdot 2x) \Rightarrow 5 = \log_2 2x^2$

* write in exponent form

$2^5 = 2x^2$

$32 = 2x^2$

$\div 2 \quad \div 2$

$16 = x^2 \Rightarrow x = \pm 4$

* $x \neq$ a negative

$\therefore x = 4$

ex 2 $\quad 2\log x - \log(x+2) = \log(2x-3)$ ← use power law

*Use quotient law

$\log \dfrac{x^2}{x+2} = \log(2x-3)$

$\dfrac{x^2}{x+2} = 2x-3 \Rightarrow x^2 = (2x-3)(x+2)$

$x^2 = 2x^2 + x - 6$

$0 = x^2 + x - 6 \Rightarrow (x-2)(x+3) \therefore x = 2$ since $x > \frac{3}{2}$ so -3 not valid

log can't be negative

* $x > 0$ and $x+2 > 0$ and $2x-3 > 0$

$\therefore x > 0, x > -2, x > \frac{3}{2}$

ex 3 $\log_6(x+3) + \log_6(x+4) = 1$

product law

$\log_6(x+3)(x+4) = 1$

$\log_6(x^2+7x+12) = 1$ * write in exponent form

$6^1 = x^2+7x+12$ * move to 1 side & factor

$-6 \qquad\qquad -6$

$0 = x^2+7x+6$

$(x+1)(x+6)$ $\boxed{\therefore x = -1 \text{ or } -6}$

* check original equation to figure out what x cannot equal (when it would be negative)

$x+3>0$

$\therefore x > -3$

$x+4>0$

$\therefore x > -4$ $\qquad x = -1$ or $\cancel{-6}$ ← invalid.

Change from exponent to log to solve.

$9^x = 50$ change to log (log both sides!)

* remember

$\log_9 9^x = x$ same

$\log_9 9^x = \log_9 50$

$x = \log_9 50$ → same as saying $\boxed{\dfrac{\log 50}{\log 9}}$

$x = \dfrac{\log 50}{\log 9}$ * calculator

$x \doteq 1.78$

5.8 Solving Problems with Exponents & Logarithms

Future Value Formula

regular investment — number of investments

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

interest

To solve for n:

$FV = 100\ 000$

$R = 100$

$i = \frac{6\%}{\text{Comp monthly}} = \frac{0.06}{12} = 0.005$

12 ← # times compounded.

$$0.005 \times 100\ 000 = \frac{100[(1+0.005)^n - 1]}{0.005} \times 0.005$$

$$500 = 100[(1+0.005)^n - 1]$$

$\div 100 \qquad \div 100$

$$5 = 1.005^n - 1 \Rightarrow 6 = 1.005^n$$

$+1 \qquad +1$

* log then * use the power law

$$\log 6 = \log 1.005^n$$

$$\log 6 = n \log 1.005$$

$\div \log 1.005 \qquad \div \log 1.005$

$$\frac{\log 6}{\log 1.005} = n$$

$$n = 359.247\ldots$$

$\therefore$ you need 360 Investments.

Present Value formula

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$PV = 200{,}000 \quad i = \frac{0.04}{12}$ - interest rate / - compounded monthly

$R = 1500$

$$200\,000 = \frac{1500\left[1-\left(1+\frac{0.04}{12}\right)^{-n}\right]}{\frac{0.04}{12}}$$

0.0033 $$200\,000 = \frac{1500\left[1-(1.0033)^{-n}\right]}{0.0033} \quad \times 0.0033$$

$$666.67 \div 1500 = 1-(1.0033)^{-n}$$

$$0.\overline{44} = 1-(1.0033)^{-n}$$

$$0.\overline{55} = (1.0033)^{-n} \quad \text{log both sides}$$

$$\log 0.55 = \log(1.0033)^{-n} \quad * \text{ use power law}$$

$$\frac{\log 0.55}{-\log 1.0033} = \frac{-n \log 1.0033}{-\log 1.0033}$$

$$+n = +176.6 \sim 177 \text{ payments.}$$

The Richter Scale

$$M = \log \frac{I}{S}$$

I ← intensity of earthquake

S ← intensity of a 'standard' earthquake.

M ← magnitude

* magnitude of an earthquake is 10^x

if an earthquake has a magnitude of 8

$$M = \log \frac{I}{S} \Rightarrow 8 = \log\left(\frac{I}{S}\right)$$ ← to exponent form

$$10^8 = \frac{I}{S} \therefore I = 10^8 S$$

or 100000000 x more powerful than a standard earthquake

** assume base 10 if not written!

To find out how much more intense an earthquake is compared to another ÷ the I's

$$M = 5 \Rightarrow I = 10^5 S$$

$$M = 9 \Rightarrow I = 10^9 S$$

$$\Rightarrow \frac{10^9}{10^5} = 10^4$$

or 10000 x more intense!