PC11 - Chapter 6 Notes
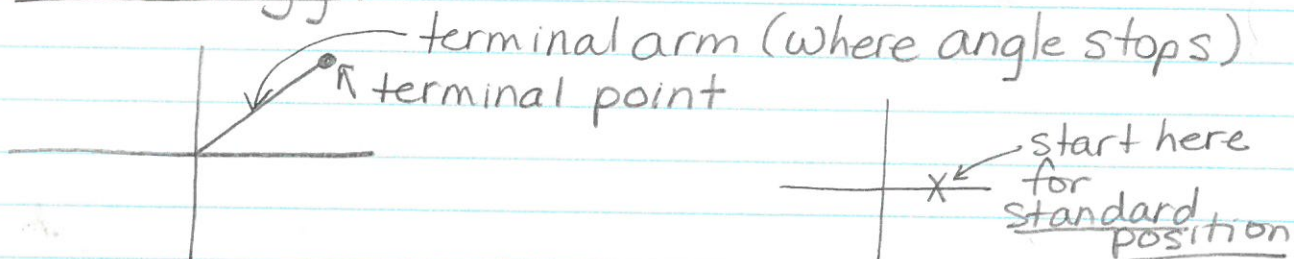
<u>Radian</u> = different unit of measure for an angle.
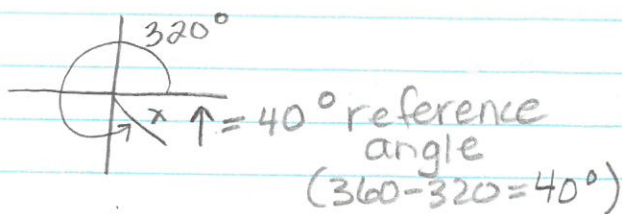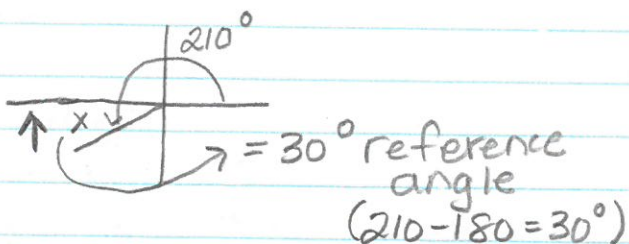
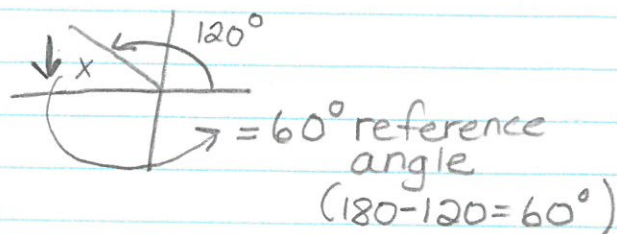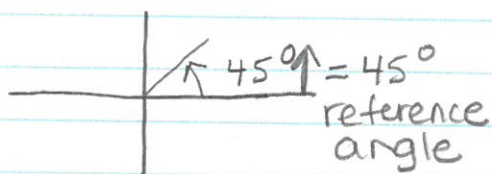$180° = \pi$ (in radians) | unit circle | ⇒ radius = 1
$360° = 2\pi$

∴ $30° = \dfrac{\pi}{6}$ (* $180° \div 6 = 30°$)

<u>Terminology:</u>

terminal arm (where angle stops)
↑ terminal point

start here
for
<u>standard</u> position

<u>Reference angle:</u> distance, in degrees, from terminal arm to <u>closest x axis</u>

$45° = 45°$
reference angle

$120°$
$= 60°$ reference angle
$(180 - 120 = 60°)$

$210°$
$= 30°$ reference angle
$(210 - 180 = 30°)$

$320°$
$= 40°$ reference angle
$(360 - 320 = 40°)$

* reference angle is <u>always positive.</u>

<u>Quadrants</u>

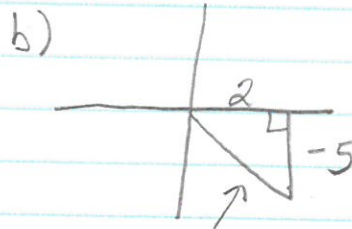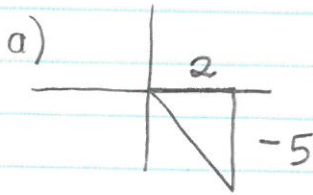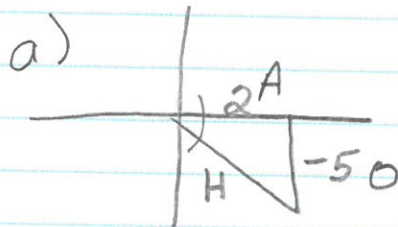| II	I
III	IV

all angles can make

H
← draw
A
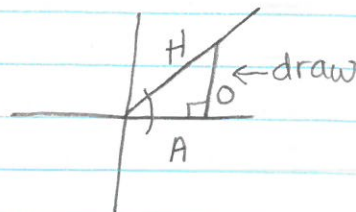
6.2

I To find $\sin\theta$, $\cos\theta$ or $\tan\theta$

① Use points to draw angle $(2, -5)$
(x, y)

a)

b)

draw diagonal (hypotenuse)
to $(0,0)$ to complete
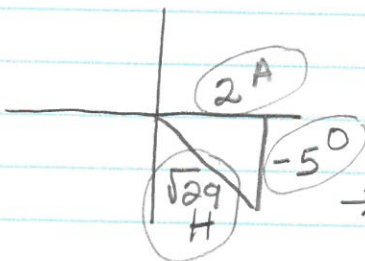
② Label O, H, A then
use Pythagorean Theorem to find H

a)

b) $a^2 + b^2 = c^2$
$2^2 + (-5)^2 = c^2$
$4 + 25 = c^2$
$\therefore c = \sqrt{29}$

③ To find SIN, COS, TAN ⟹

$$\sin\theta = \frac{O}{H}$$
$$\cos\theta = \frac{A}{H}$$
$$\tan\theta = \frac{O}{A}$$

$\to \sin\theta = \frac{-5}{\sqrt{29}}$

$\cos\theta = \frac{2}{\sqrt{29}}$

$\tan\theta = \frac{-5}{2}$

④ To find the angle (✻ make sure calculator
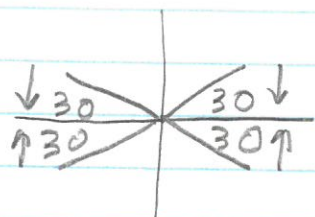is on degrees)
use any of the trig values
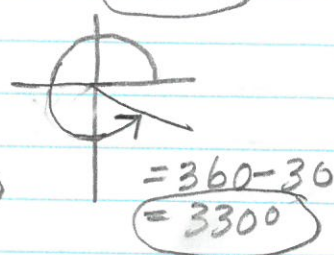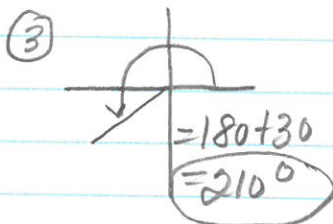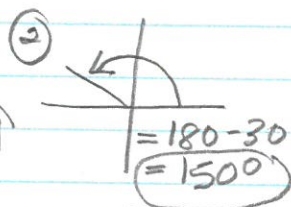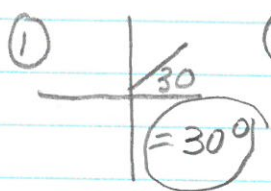+ <u>shift</u>   ex. shift $\sin^{-1}(-5 \div \sqrt{29}) =$

II To find other angles with the same reference angle:

* note: $0° \leq \theta \leq 360°$
means find angles between 0° and 360°

ex reference angle $= 30°$

$\downarrow 30$ | $30 \downarrow$
$\uparrow 30$ | $30 \uparrow$

possible angles between 0° and 360°

① $\boxed{= 30°}$ $\quad$ 30

② $= 180 - 30$
$\boxed{= 150°}$

③ $= 180 + 30$
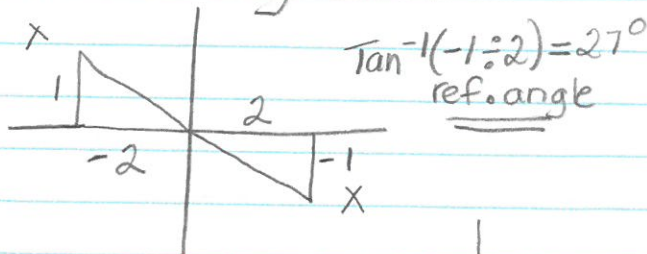$\boxed{= 210°}$ $\qquad$ $= 360 - 30$
$\boxed{= 330°}$

III To find possible angles using one trig ratio.

① check Quadrants to determine where it could be. $\qquad$ ex. $\tan \theta = -\frac{1}{2} = \frac{O}{A}$
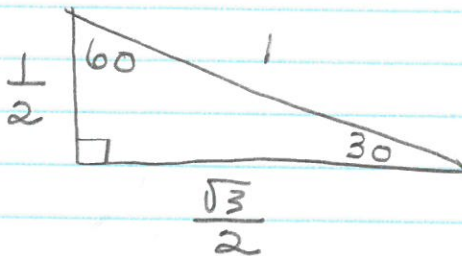
SIN + (cos and Tan −)	All +
(sin and cos −) TAN +	cos + (Tan and sin −)

"All Students take Calculus"

② Draw using sides

$\tan^{-1}(-1 \div 2) = 27°$
ref. angle

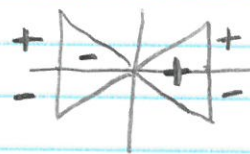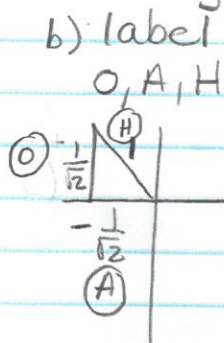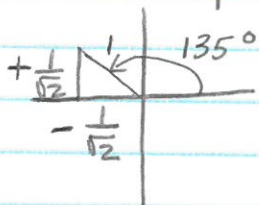③ $= 180 - 27 \Rightarrow \boxed{153°}$ $\qquad$ and $\qquad$ $= 360 - 27 \Rightarrow \boxed{333°}$
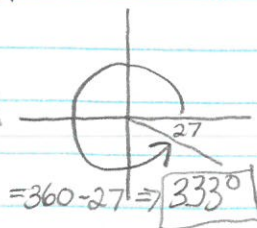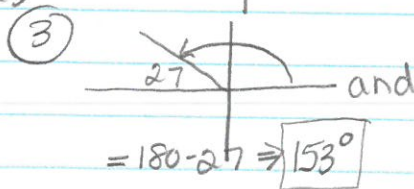
IV To find Exact Value of trig ratio

① To find any angle with a reference angle of: $30°, 45°$ or $60°$ use the 2 special triangles
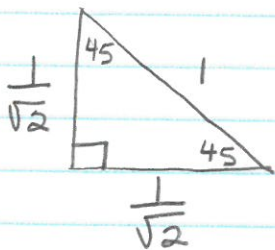
ex find exact value of SIN 135

a) draw special triangle (remember to add + or −)

b) label O, A, H
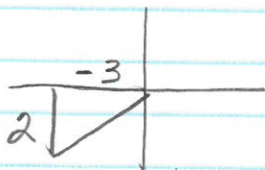

c) $SIN 135 = \frac{O}{H}$

$= \frac{1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}$
$\qquad\qquad\quad \overline{1}$

② Exact value using a point

ex. P(−3, 2)

① Draw in approriate quadrant

$(-,+) \mid (+,+)$ $SIN \theta = \frac{O}{H}$
$\overline{\qquad\qquad}$ $COS \theta = \frac{A}{H}$
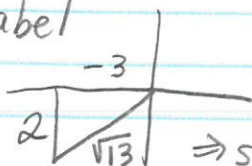$(-,-) \mid (+,-)$ $Tan \theta = \frac{O}{A}$

② use pythagorean Theorem
$a^2 + b^2 = c^2$
$(2)^2 + (-3)^2 = c^2$
$13 = c^2$
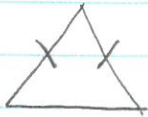∴ $c = \sqrt{13}$ (for Exact value *leave √)

③ Label

$\Rightarrow SIN \theta = \frac{2}{\sqrt{13}}$, $COS \theta = \frac{-3}{\sqrt{13}}$, $Tan \theta = \frac{2}{-3}$

6.3

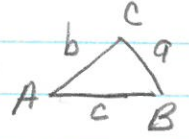180 = sum of all angles in a triangle = $180°$

Types of triangles:

Isosceles = 2 sides are equal ∴ ∝ angles equal

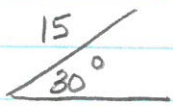equalateral → all sides and angles equal

right angle
- has one angle at $90°$
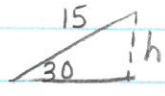- can use Pythagorean Theorem to find other sides and angles

scalene → not any of the other △'s
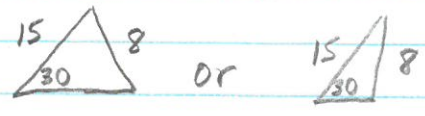
To determine how many triangles : $A \angle \overset{b \quad c \quad a}{\underset{c \quad B}{}}$

① draw angle

② label angle and side

③ use $\sin \theta = \frac{O}{H}$ to find o
- this will be the height if a △

ex 1
$A = 30°$
$b = 15$
$a = 8$

$A \angle$

15 / $30°$

15 /|h /30

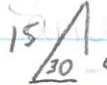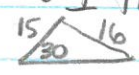$\sin 30 = \frac{x}{15}$
$(\sin 30) \times 15 = x$
$x = 7.5$
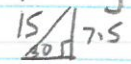
∴ you could make 2 triangle.

15 /8\ /30 or 15 /|8 /30
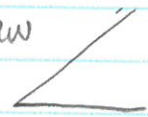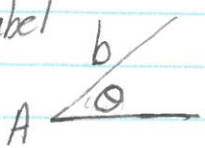
ex 2
$A = 30°$ → ∴ no triangles possible
$b = 15$
$a = 6$

15 /| /30 ← doesn't reach!

ex 3
$A = 30$
$b = 15$
$d = 16$

∴ you could make 1 triange
15 /16 /30 /| ← too long

ex 4
$A = 30$
$b = 15$
$d = 7.5$

∴ you could make 1 triange (a right angle)
15 /|7.5 /30
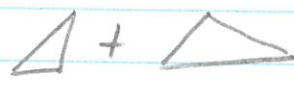
To determine how many triangles: (Summary)

① draw

② label

b

$A \angle \theta$

③ use $SIN\theta = \frac{x}{given\ side}$

$x = min.\ height$

rules

∅ trianges ⇒ if given side is < x $\angle$

1 triange ⇒ a) if given side is = x

b) if given side is > x and > b

2 triangles ⇒ if given side is > x and < b

Review: right angled Trig   *make sure calculator is on DEG

* can <u>only</u> be used with a <u>right angle triangle</u>

$SIN\theta = \frac{O}{H}$

$COS\theta = \frac{A}{H}$

$TAN\theta = \frac{O}{A}$
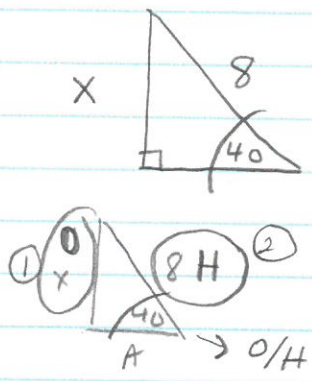
x 8   40°

① label O, A, H

② circle what you want + have to choose formula

③ fill in formula + rearrange to solve for x

O/H ∴ $SIN\theta = \frac{O}{H}$

③ $SIN40 = \frac{x}{8}$ ⇒ $(SIN40) \times 8 = x$

$x = 5.1$

* remember to find angle use shift.

ex $Tan\ x = \frac{4}{3}$

= shift tan ⇒ $tan^{-1}(4 \div 3) = 53°$

6.4 SINE LAW   *use if not a right triangle

Sine Law = ratio of angles to sides
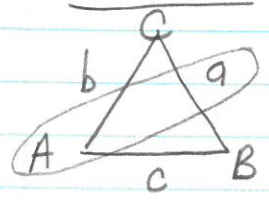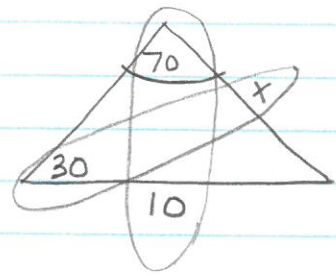
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

*use the sin law when you have one side and angle pair
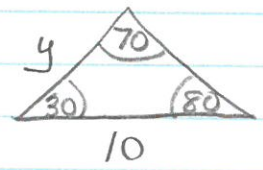
ex

SIDE X

$$\frac{10}{\sin 70} = \frac{x}{\sin 30} \qquad \text{solve for } x$$

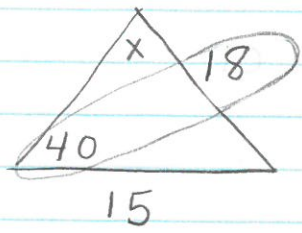$$\frac{10 \times \sin 30}{\sin 70} = 5.3$$

* remember - you can solve for other side by $180 - 30 - 70 = 80$

$$\frac{10}{\sin 70} = \frac{y}{\sin 80} \qquad \text{solve for } y$$

$$\frac{10 \times \sin 80}{\sin 70} = 10.5$$

ANGLE X

$$\frac{18}{\sin 40} = \frac{15}{\sin x} \qquad \text{Cross } x \text{ and } \div$$

$$\frac{\sin 40 \times 15}{18} = 0.536$$

* now shift SIN (0.536)

$$x° \sim 32°$$

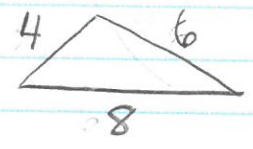**fun fact** $x \quad y$ Supplementary angle.
$$\Rightarrow x + y = 180$$

$$SIN \, 30 = SIN \, 150 \qquad SIN \, 60 = SIN \, 120 \qquad SIN \, 135 = SIN \, 45 \ldots$$
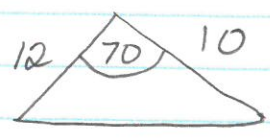
* SIN (any Supplementary angles) are =

6.5 COSINE LAW

*use if you have the following situations:

① you are given all 3 sides but no angles
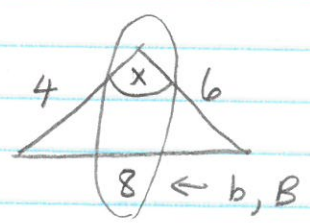
② you are given 2 sides beside an angle.

12 /70\ 10

** no matter what the triangle is labelled, b and B are the across pair of what you want.

Cosine Law: $b^2 = a^2 + c^2 - 2ac \cos B$

* across from each other
* one of them is x

angle ⓘ using side

ex 1

4 /x\ 6

8 ← b, B

$b^2 = a^2 + c^2 - 2ac \cos B$
$8^2 = 4^2 + 6^2 - 2(4)(6) \cos x$
$64 = 52 - 48 \cos x$ * use SAMDEB
$-52 \quad -52$

* careful when solving for Cos x
$52 - 48 \cos x$
→ x cannot put together *

$\dfrac{12}{-48} = \dfrac{-48 \cos x}{-48}$

$-0.25 = \cos x \Rightarrow \text{shift} \cos^{-1}(-0.25)$
$= 104°$

Now use SINE LAW to solve the rest:

ⓘ $\dfrac{8}{\sin 104} = \dfrac{4}{\sin x}$

ⓘⓘ $180 - 29 - 104 = 47°$ (missing angle)

4 /104\ 6
/47   29\
8

$\dfrac{4 \times \sin 104}{8} = \sin x$

$x = 29°$
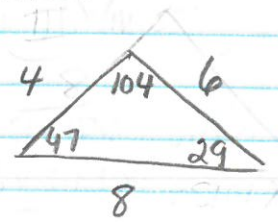
Ⅱ COSINE LAW: find side w̄ angle

side using angle

E
12 /70\ 10
D / X \ F
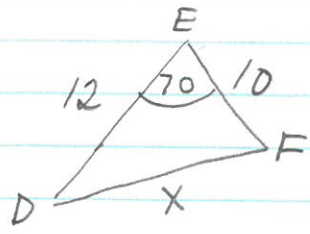
* remember angle/side pair you want is always B, b

B
E
12 /70\ 10
b
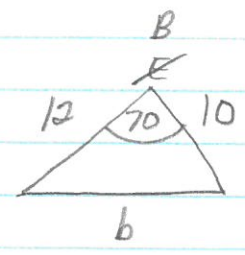
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$b^2 = 12^2 + 10^2 - 2(12)(10) \cos 70$$
$$= 244 - 82.08$$
$$b^2 = 161.92$$
$$b = 12.7$$

** hint put like this in calculator
$-2 \times 12 \times 10 \times \cos 70$
so you dont forget the negative!

Now use SINE LAW to solve the rest

12 /70\ 10
X \ 12.7

$$\frac{12.7}{\sin 70} = \frac{10}{\sin X}$$

$$\frac{(\sin 70) \times 10}{12.7} = \sin X \qquad X = 48°$$
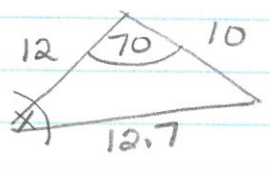
FINALLY

Sum 180 → $180 - 70 - 48 = 62°$

12 /70\ 10
48 \ 62
12.7

TA - DA!