Foundations 11

Class notes: Chapter 1

1.1 Conjecture: a testable idea/expression that is not based on evidence.

inductive reasoning: conclusion based on patterns in specific examples

deductive reasoning: conclusion based on valid general assumptions.

When developing a conjecture:

→ 2 even integers (sum)

1. do some examples $2+4=6$ $4+6=10$
2. write a statement (adding 2 even integers = even)
3. test it out with other examples.
   $10+12=22$ $4+8=12$
4. use inductive reasoning to explain conjecture.

1.3 Explore validity of conjectures / Counterexample.

Counterexample - an example that will disprove the conjecture.

ex. Conjecture = difference of 2 square number is a prime number

$2^2-1^2=4-1=3$ ✓

$3^2-2^2=9-4=5$ ✓

$5^2-4^2=25-16=9$ ← counter example
9 is not a prime number

$\boxed{1.4}$ Deductive Reasoning:

proof: no counter example exists

generalization - statement made using some proof to make an application of these ideas.

deductive reasoning: making a specific statement using general assumptions that are valid.

→ using deductive reasoning, you try to create a rule that would apply to all situations.

ex. Difference between consecutive perfect square numbers is always odd

$26^2 - 25^2 = 51$ make $x = 25$

$\therefore 26 = x + 1$

$(x+1)^2 + x^2 \Rightarrow (x+1)(x+1) - x^2$

$= x^2 + 2x + 1 - x^2$

$= 2x + 1$

it will be odd since

$2 \times$ a number = even

even + 1 = odd.

$\boxed{1.5}$ Invalid Proofs:

<u>invalid proof</u>: one that can be proven wrong or contains incorrect assumptions

<u>ex.</u> Athletes do not compete in both Summer and Winter Olympics.
true: for those who play one sport.
Counterexample → false: for those who play more than one sport

∴ this conjecture is invalid.

<u>ex</u> $3=4$

proof: $a+b=c$

$$4a-3a+4b-3b=4c-3c$$
$$4a+4b-4c=3a+3b-3c$$
$$4(a+b-c)=3(a+b-c) \quad \div \text{ by } (a+b-c)$$
$$4=3$$

Counterexample
Proof that proof is invalid

$$a+b=c$$
$$a+b-c=c-c$$
$$a+b-c=0 \quad * \text{ cannot } \div \text{ by } 0$$

So $\frac{4(a+b-c)}{a+b-c} = \frac{3(a+b-c)}{(a+b-c)}$

→ is undefined & not a valid proof.

1.6 When choosing between inductive & deductive reasoning

<u>inductive</u>: Solves simpler problems
looks for patterns
Small → big idea then makes a statement

<u>deductive</u>: uses known facts / assumptions
to make a statement
big idea → small then solves the problem

* when asking if statement / conjecture is valid - look for flaws

<u>ex</u> mammals have hair ← * do <u>all</u> mammals have hair?
Dogs are mammals
∴ Dog have hair.
This is an example of deductive reasoning.

<u>ex</u> Every even number has a factor of 2

2 = 1, 2   4 = 1, 2, 4   12 = 1, 2, 3, 4, 6, 12

∴ 24 has a factor of 2.

This is an example of inductive reasoning.

1.7 Games

<u>inductive</u> - good for games with patterns / order

<u>deductive</u> - good for games with inquiry / discovery