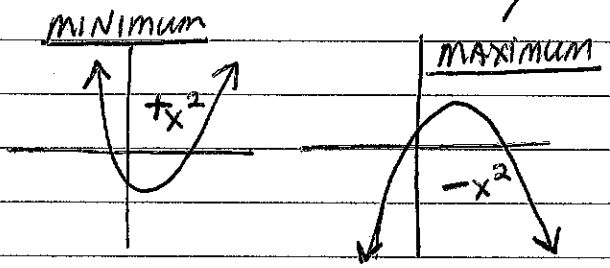
7.1 Quadratic Functions

Standard form ⇒ $y = ax^2 + bx + c$

parabola – shape made when graphing a quadratic function

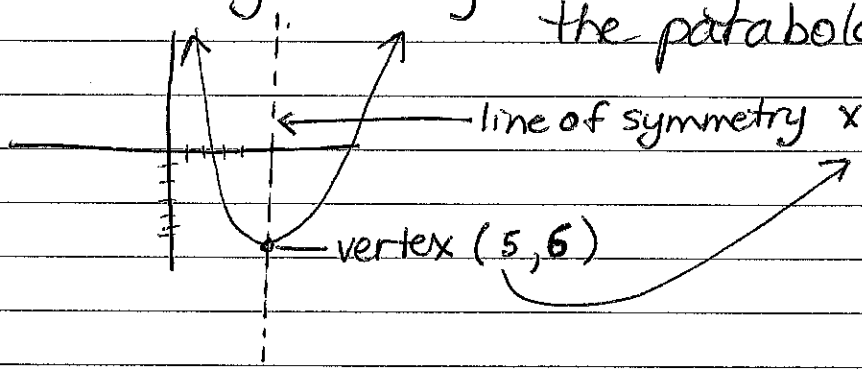


opens up when x^2 is +

opens down when x^2 is −

degree = highest exponent → of a quadratic = 2

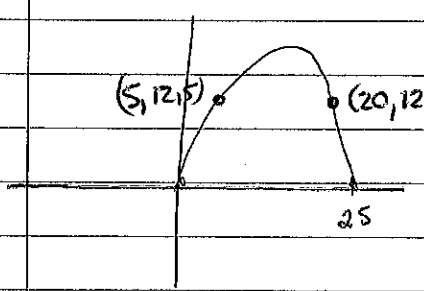
line of symmetry – exactly half way across the parabola.



7.2

vertex – the maximum or minimum point on the graph

To find the vertex find 2 points on the
① graph with the same y value.



(0,0) (25,0) or (5, 12.5) (20, 12.5)

② add x values & divide by 2

$\frac{5+20}{2} = \frac{25}{2} = 12.5$ x value of vertex & line of symmetry

To find the y value of the vertex.

Take the x value you just calculated & plug it into the equation

$x = 12.5$ equation $\Rightarrow f(x) = -0.12x^2 + 3x$

plug in: $f(12.5) = -0.12(12.5)^2 + 3(12.5)$

$= -0.12(156.25) + 37.5$

$= -18.75 + 37.5$

$= +18.75$

$V(12.5, 18.75)$

NOTE: $f(x) =$ means the same thing as $y =$

ex 2 To find the vertex:

$y = x^2 + x - 2$

① Make a table of values

x	y	
0	-2	$(0)^2 + (0) - 2$
1	0	$(1)^2 + (1) - 2$ ←
-1	-2	$(-1)^2 + (-1) - 2$
2	4	$(2)^2 + (2) - 2$
-2	0	$(-2)^2 + 2 - 2$ ←

the y value is the same

② So add x values & divide by 2

$\frac{1 + -2}{2} = -0.5$

③ plug -0.5 into the equation

$y = (-0.5)^2 + (-0.5) - 2$

$y = -2.25$

$V(-0.5, -2.25)$

7.3 Solving Quadratic Equations by graphing

When using the graphing calculator

Method 1 – one equation t = time, y = height

$y = 5.0 + 24.46t - 4.9t^2$

$y = 4$ } make equal to each other

$4 = 5.0 + 24.46t - 4.9t^2$

$-4 \quad -4$

rearrange so it reads

$0 = 1 + 24.46t - 4.9t^2$

(t^2 is the same as x^2 when using calculator)

use [y=] button & type in equation

Press graph → the answer/solution is the x-intercept when y=0

There are 2 answers −0.041 or 4

since time can't be negative

$t = 4$

Method 2

→ plug in both equations & find where they cross

[y=] $5.0 + 24.46t - 4.9t^2$

[y=] 4

[2nF] CALC: INTERSECTION

* you'll get same answers as above

7.4 Factored form of a Quadratic Function

You can find the x intercepts by factoring.

x-intercepts are also called zeros, roots & Solution

$y = 2x^2 + 14x + 12$

① factor out GCF all can ÷ by 2

$2(x^2 + 7x + 6)$

list factors:

1×6

2×3 → ask which set of factors could combine to make 7.

$1 + 6 = 7$ So use them

$2(x+1)(x+6)$

x intercepts - Ask what would make ( ) = 0

$(x+1) = 0 \therefore x = -1$

$(x+6) = 0 \therefore x = -6$

x intercepts

$(-1, 0)(-6, 0)$

y intercept - make $x = 0$

$y = 2(0)^2 + 14(0) + 12$

$= 12$ y intercept = $(0, 12)$

Open up or down - look in front of x^2

$y = 2x^2 + 14x + 12$ it is positive so opens up.

To find the vertex

Take x intercepts & ÷ by 2

$$\frac{-1 + -6}{2} = \frac{-7}{2} = -3.5$$

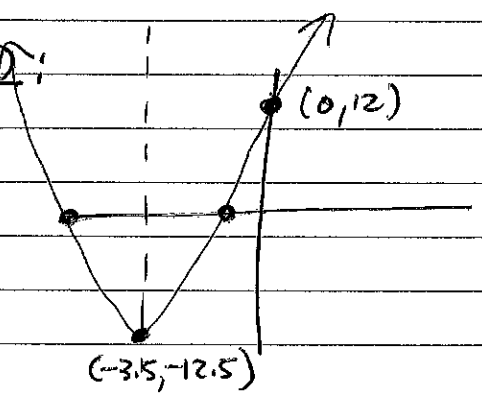
then plug into equation:

$$y = 2(-3.5+1)(-3.5+6)$$
$$= 2(-2.5)(2.5)$$
$$= -12.5$$

$V(-3.5, -12.5)$

Now graph all you know:

x intercepts $(-1,0)$ $(-6,0)$
y intercept $(0,12)$
Vertex $(-3.5, -12.5)$
opens up
line of symmetry $\Rightarrow x = -3.5$



Find equation using the graph:

Use x intercepts & plug into this $y = a(x-r)(x-p)$

if x intercepts are $(-2,0)$ $(6,0)$; $P(2,8)$ (x, y)

$$y = a(x-(-2))(x-6)$$
$$y = a(x+2)(x-6)$$

now use any other point from the graph for x & y values

$$8 = a(2+2)(2-6)$$
$$8 = a(4)(4)$$
$$8 = 16a \quad a = \frac{8}{16} = \frac{1}{2}$$

$$y = \frac{1}{2}(x+2)(x-6)$$
$$y = \frac{1}{2}(x^2 - 4x - 12)$$

To return an equation into standard form use distributive property

$$y = \frac{1}{2}(x+2)(x-6)$$

$$= \frac{1}{2}(x^2 - 6x + 2x - 12)$$

$$= \frac{1}{2}(x^2 - 4x - 12)$$

$$y = \frac{1}{2}x^2 - 2x - 6$$

Revenue problems ~~x~~ find max ie vertex

R = (# of plates)(price)

Fries sell for \$3, for every \$1 increase they sell 20 fewer plates of fries

They usually sell 200 plates

R = (# plates)(price)

$(200 - 20x)(3 + \$1x)$

x intercepts

$200 - 20x = 0$ or $3 + x = 0$

$200 = 20x$ $x = -3$

$x = 10$

x intercepts are 10 & -3 ⇒ add & ÷ 2

$\frac{10 + -3}{2} = 3.5$ so max increase is \$3.50

or a price of \$6.50

Max revenue is $(200 - 20(3.50))(3 + 3.50) = \$845$

130×6.50

7.5 Solving Quadratic Equations by Factoring

You are given an equation

$$h(w) = -0.625w^2 + 5w$$

Determine the distance between 2 points on an arch that are 7.5 ft high.

① make $h(w) = 7.5$

$\rightarrow$ $7.5 = -0.625w^2 + 5w$

② re-arrange so everything is on one side

$$7.5 = -0.625w^2 + 5w$$
$$+0.625w^2 \quad +0.625w^2$$

$$0.625w^2 + 7.5 = 5w$$
$$-5w \quad -5w$$

$\rightarrow$ $0.625w^2 - 5w + 7.5 = 0$

③ $\div$ everything by what is infront of w^2

$$\frac{0.625w^2}{0.625} - \frac{5w}{0.625} + \frac{7.5}{0.625} = 0$$

$$0.625(w^2 - 8w + 12) = 0$$

factor $0.625(w-2)(w-6) = 0$

$\therefore$ $w = 2, 6$ at 7.5 ft

Distance between $\Rightarrow 6 - 2 = 4$ ft

7.6 Vertex form of a Quadratic Function

$$f(x) = a(x-p)^2 + q$$

(vertex)

Vertex:

ex $f(x) = -2(x-3)^2 + 4$ (opens down)

$V(3,4)$

axis of Symmetry $x = 3$

y-intercept: make $x = 0$

$y = -2(0-3)^2 + 4$

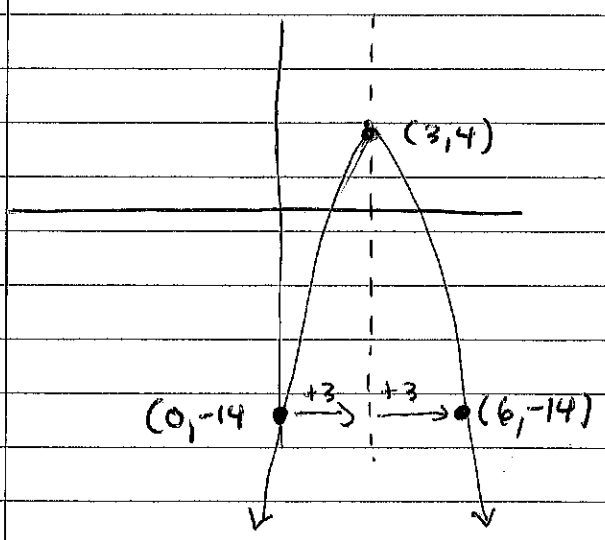
$y = -2(-3)^2 + 4$

$y = -2(9) + 4$

$y = -18 + 4$

$y = -14$

y-intercept $(0, -14)$



$f(x) = 3(x+2)^2 - 3$ (opens up)

$V(-2,-3)$

$x = -2$

$y = 2(0+2)^2 - 3$

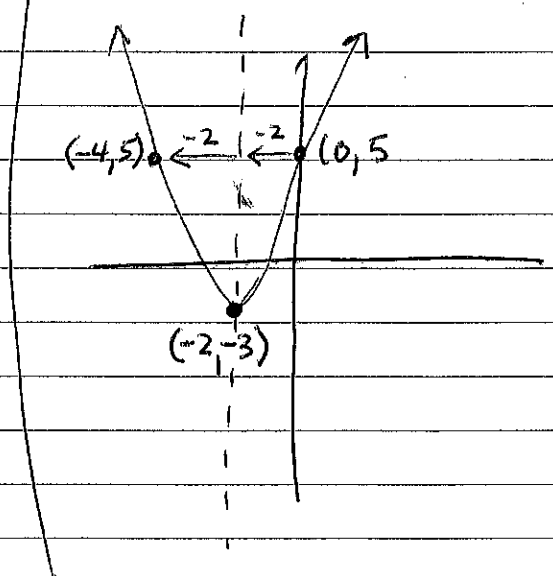
$= 2(2)^2 - 3$

$= 2(4) - 3$

$= 8 - 3$

$y = 5$

y-intercept $(0, 5)$



Finding the equation using vertex:

$y = a(x-p)^2 + q$

Vertex $(12, 48) \Rightarrow$ plugin $\Rightarrow$ $y = a(x-12)^2 + 48$

$P(10, 85)$ → put point in for x + y value

$85 = a(10-12)^2 + 48$

Solve for a

$85 = a(-2)^2 + 48$

$-48 \qquad -48$

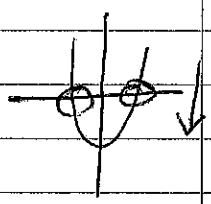
$37 = 4a \quad \div 4$

$\frac{37}{4} = a \Rightarrow 9.25 = a$

Now write equation:

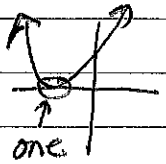
$y = 9.25(x-12)^2 + 48$

* Determining number of zeros: (x-intercepts)



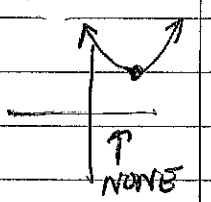
$f(x) = 2(x-2)^2 - 5$ ← goes down 5; so I think 2 roots

Check w calculator → yes 2 roots



$f(x) = 2(x+3)^2$ ← 0 up or down so 1 root

Check - yes 1 root



$f(x) = 2(x-3)^2 + 5$ ← up 5 ∴ no roots

Check - No roots

7.7 The Quadratic Formula

$y = ax^2 + bx + c \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

① make sure the equation is in the same order as above
- if anything is missing add a 0

ex $2 = 2x^2 - x \Rightarrow 0 = 2x^2 - x - 2$

$4x = 7x^2 \Rightarrow 0 = 7x^2 - 4x + 0$

$3 = 2x^2 \Rightarrow 0 = 2x^2 + 0x - 3$

② plug into the equation (the answer will give you x intercepts)

$y = 2x^2 - 3x + 1$
$\downarrow \quad 1 \quad 1$
$a \quad b \quad c$

$a = 2$
$b = -3$
$c = 1$

* dont forget to include + or −

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)}$

③ $= \frac{3 \pm \sqrt{9 - 8}}{4}$

now really 2 equations

$\frac{3 + \sqrt{1}}{4} = \frac{4}{4} = 1$

$\frac{3 - \sqrt{1}}{4} = \frac{2}{4} = \frac{1}{2}$

④ x intercepts: 1 and $\frac{1}{2}$

Leaving answer in exact form

$2x^2 + 8x - 5 = 0$

$a = 2 \quad b = 8 \quad c = -5$

plug in $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-5)}}{2(2)}$

$= \frac{-8 \pm \sqrt{64 - (-40)}}{4}$

$= \frac{-8 \pm \sqrt{64 + 40}}{4}$

exact form $= \frac{-8 \pm \sqrt{104}}{4}$

* now reduce $\left(\sqrt{104} = 2\sqrt{26} \right)$

$104 = 4 \cdot 26$

$4 = 2 \cdot 2 \quad 26 = 2 \cdot 13$

$= \frac{-8 \pm 2\sqrt{26}}{4}$ ÷ all whole # by 2

$= \frac{-4 \pm \sqrt{26}}{2}$

$\frac{-4 + \sqrt{26}}{2}$ or $\frac{-4 - \sqrt{26}}{2}$