Chapter 3 notes - Math 11 PC

Review Factoring:

Method 1

$y = 2x^2 + 6x - 8$

① multiply

② Prod: $2 \times -8 = -16$ | Sum: $+6$

list factors: 1×-16, 16×-1, 2×-8, 8×-2, 4×-4

$8 + -2 = 6$ ← which ones add up to middle #

③ Substitute factors in for middle number $2x^2 + 8x - 2x - 8$

④ factor

$2x^2 + 8x - 2x - 8$

$= 2x(x+4) - 2(x+4)$

$= (2x-2)(x+4)$

⑤ remove common factor

$= 2(x-1)(x+4)$

Method 2

① factor 1st !!

$y = 2x^2 + 6x - 8 \Rightarrow y = 2(x^2 + 3x - 4)$

② list factors: 1×4, 2×2 ← ask → which 2 can combine to make middle #?

③ place factors in brackets

$= 2(x \quad 1)(x \quad 4)$

$1 + 4 = 3$ ← which one needs to be negative to make $= 3$

$= 2(x-1)(x+4)$

ex #2 if x^2 has a co-efficient that cannot be factored away.

$y = 3x^2 + 7x + 4$ (no common factors ☺)

① find factors of front & back: $3 \to 1, 3$; $4 \to 1, 4$ or $2, 2$

② plug in & guess & check to make middle #

$(3x \;\; 1)(x \;\; 4)$ → +12 → NO, too big

$(3x+4)(x+1)$ → +4, +3 → yes

Perfect squares

ex $9b^2 + 12b + 4$ ← middle # will be $\sqrt{}$ of front $\times$ $\sqrt{}$ back $\times 2$

$(3b+2)(3b+2)$

+ if middle # is +

Difference of squares (no middle # but front & back are perfect squares)

ex $49x^2 - 81$

$\sqrt{}$ $\sqrt{}$

① square root both front & back,

② place in brackets & put +, − in

$(7x+9)(7x-9)$

3.1

Decimals / Fractions!

$y = x^2 + 1.4x - 1.2$ ① * get rid of decimals by ~~xxxx~~ ÷ 0.1 (same as x 10)

$y = 0.1(10x^2 + 14x - 12)$ $\frac{1}{0.1} = 10$

② factor out common factors

$y = 0.1(2(5x^2 + 7x - 6)$

③ factor remaining trinomial

$y = 0.2(5x - 3)(x + 2)$

To simplify ✓ in equation ( )² both sides

Questions with brackets

⊗ Substitute ... then plug back in

ex1 same bracket

$y = (x-3)^2 - 6(x-3) - 16 \Rightarrow$ ① $a = (x-3)$

② $y = a^2 - 6a - 16$ factor

$y = (a+2)(a-8)$

③ plug back in

$y = ((x-3)+2)((x-3)-8)$

$= (x-1)(x-11)$

ex2 different

$(2x-1)^2 - 9(y+4)^2 \Rightarrow$ ① $a = (2x-1)$, $b = (y+4)$

② $a^2 - 9b^2$ factor

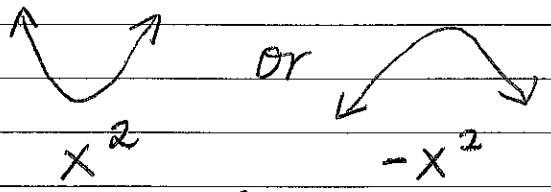
$(a+3b)(a-3b)$

③ sub back in $((2x-1)+3(y+4))((2x-1)-3(y+4)) = (2x+3y+11)(2x-3y-13)$ (3y+12, -3y-12)

3.2

Factoring & roots

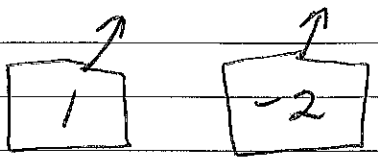
Any quadratic equation has a shape like this:



When you factor → you find the x-intercepts (also known as zeros or roots)

ex 1 $y = x^2 + x - 2$

factored $y = (x-1)(x+2)$ * ask what would make the bracket = 0 these are your x intercepts or roots.

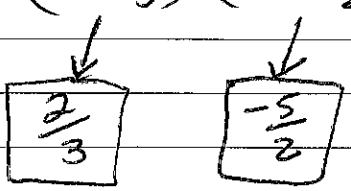


ex 2 fractions

$y = (3x-2)(2x+5)$ ← ÷ both terms by # in front of bracket

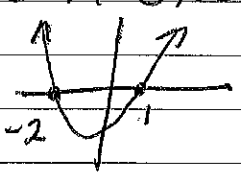
$= \left(\frac{3x}{3} - \frac{2}{3}\right)\left(\frac{2x}{2} + \frac{5}{2}\right)$ simplify

$= \left(x - \frac{2}{3}\right)\left(x + \frac{5}{2}\right)$ roots → make bracket = 0

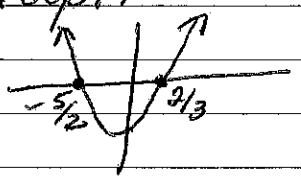


Once you find the roots / x intercept, you can sketch the graph

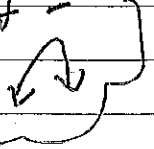
ex 1



ex 2

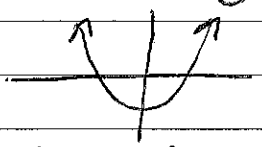


* if -



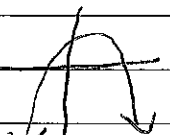
General shapes of graphs

2 different roots



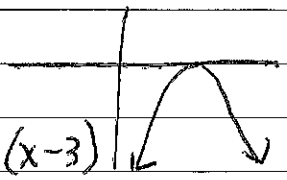
$(x+3)(x-2)$

or if $-x^2$

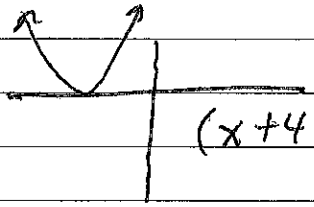


$-(x+1)(x-3)$

2 same roots

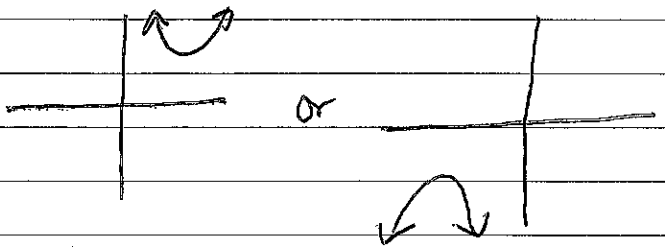


$-(x-3)(x-3)$



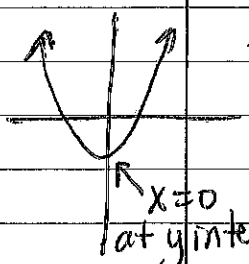
$(x+4)(x+4)$

no roots (ie. can't factor)
→ doesn't cross x axis



y intercept

to complete a more accurate sketch find the y-intercept by making $x=0$



use $y = x^2 + x - 2$

x intercepts ⇒ factor

$y = (x-1)(x+2)$

x intercepts $(1,0)$ $(-2,0)$

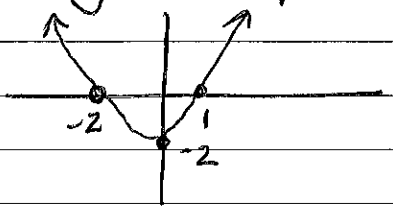
y intercept $x = 0$

$y = 0^2 + 0 - 2$

$y = -2$

y intercept ⇒ $(0,-2)$

sketch



The quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow$$

View the equation you need to solve as:

$$\underline{a}x^2 + \underline{b}x + \underline{c} = 0$$

ex $3x^2 + x - 2 = 0$
[a] [b] [c]

* remember to keep the sign with the #.
* If the is no # - then it is 1.

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(1) \pm \sqrt{1^2-4(3)(-2)}}{2(3)}$$

$$= \frac{-1 \pm \sqrt{1+24}}{6}$$

$$= \frac{-1+\sqrt{25}}{6} \quad \text{and} \quad = \frac{-1-\sqrt{25}}{6}$$

$$= \frac{4}{6} = \frac{2}{3} \qquad = \frac{-6}{6} = -1$$

∴ the roots are $\frac{2}{3}$ and -1

or $(\frac{2}{3}, 0)$ and $(-1, 0)$

(A) * if the number under the "$\sqrt{}$" doesn't factor evenly → ① change it to a decimal with your calculator
② keep in radical form.

(B) * ~~if the number under "$\sqrt{}$" is a negative~~ there are no real roots.

ex A $x^2 - 6x + 7 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 28}}{2}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= \frac{6 + \sqrt{8}}{2} \qquad = \frac{6 - \sqrt{8}}{2}$$

* $\sqrt{8} = \sqrt{2 \times 2 \times 2} = 2\sqrt{2}$

reduce $\frac{6 + 2\sqrt{2}}{2}$

$= 3 + \sqrt{2}$ and $3 - \sqrt{2}$

ex B $x^2 - 5x + 7 = 0$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 - 28}}{2} \qquad = \frac{5 \pm \sqrt{-3}}{2}$$ ← cannot be negative

no real roots

Things to remember when completing the square.

1. only use the 1st 2 terms to start
2. factor out number infront of x^2
3. take ½ of the 2nd number (on x) and
   a) place inside bracket
   b) take negative outside of bracket and square it.
   c) remember to multiply negative from b by number factored from x^2 term.
4. Combine (c) with number on the outside of bracket.

ex. without factoring

$x^2+2x-46=2$

re write only 1st 2 terms

$x^2+2x-48=0$

$\boxed{x^2+2x}-48=0$

$\left(x^2+\frac{2^2}{2}-\frac{2^2}{2}\right)-48=0$

$(x+1)^2-1^2-48=0$

same sign as original equation

combine

$(x+1)^2-49=0$

factoring required.

$5x^2-20x-65=-5$

$5x^2-20x-60=0$ ← re write

$5(x^2-4x)-60=0$ ← Factor 5 out of 1st 2 terms

$5\left(x^2+\frac{4^2}{2}-\frac{4^2}{2}\right)-60=0$

$\times 5$

$5(x-2)^2-(5)(2^2)-60=0$

$5(x-2)^2-20-60=0$

$5(x-2)^2-80=0$

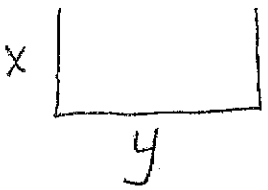
Max/Min Problems

Method 1:

* hint: draw it out & label sides x & y

① write the equations from the word problem
② re-arrange one of the equations to find max/min ← plug into max/min equat°.
③ expand
④ complete the square
⑤ $a(x-p)+q \Rightarrow -p$ value is answer for x, q is the max/min

ex. A rectangular lot has one side along a river. The other 3 sides have a total of 80m of fencing. What is the max area?



① equation #1 $2x+y=80m$

max equation → $xy=max$

② $2x+y=80 \Rightarrow 2x-80=-y \Rightarrow -2x+80=y$

$xy=max$; $x(-2x+80)=max$

③ $-2x^2+80x=max$

④ $-2(x^2-40x)$; $-2(x^2-(20)^2+20^2)$;
$-2(x-20)^2-20^2x-2$

$=-2(x-20)^2+800$

x value ↖ max area

∴ the max area is $800m^2$
the x value (width) is 20
So the y value (length) $= 2(20)+y=80$; $y=40$.

Max/Min Problems:

Method 2 → use when trying to find revenue when # increases or decreases because of price.

$R = (\text{cost})(\text{number})$

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ex. Computer company sells software to students for $20. Three hundred students buy the software. For every $5 increase in price, 30 less students buy the software.

Find max revenue & max price.

① $R = (\underset{\text{orig. cost}}{20} + x)(\underset{\text{orig. \# sold}}{300} - \underset{\text{\# less}}{30}(\frac{x}{5}))$ ← per $5 increase

② expand ⇒ $(20+x)(300-6x)$

$= -6x^2 + 180x + 6000$

③ complete the square ⇒ $-6(x^2-30x) + 6000$

$-6(x^2-15^2+15^2) + 6000$

$-6(x-15)^2 + 6000 - 15^2 x - 6$

$= -6(x-15)^2 + 7350$

↑ increase to get max revenue

↑ max revenue

∴ price for max revenue is $\$20 + \$15 = \$35$.