Math 12F - notes

Chapter 4

4.1

Ⓐ Fundimental Counting Principle - ← use when disjointed sets

if there are x ways to do the first thing and y ways to do the next thing then total ways is $x \cdot y$

ex ① number of different combination for a lock with 3 numbers - each with 60 choices (repeats allowed)

$\_\_\_ \cdot \_\_\_ \cdot \_\_\_ = 60 \times 60 \times 60$

$= \cancel{2140000}$ combinations
216000

② same as above - but no repeats allowed

$60 \cdot 59 \cdot 58 = 205320$ combinations

Ⓑ Examples that $\boxed{\text{Cant use Fund. Count. Principle}}$

a) draw a red face card or a 9.
- break it up into steps

Ⓡ draw a red face card
- 3 face cards in each suit
- 2 of these are red.
= 6

Ⓝ draw a 9
- 1 9 in each suit
- 4 suits
= 4

$R = \{Q\heartsuit, Q\diamondsuit, K\heartsuit, K\diamondsuit, J\heartsuit, J\diamondsuit\}$

$n(R) = 6$

$N = \{9\heartsuit, 9\diamondsuit, 9\clubsuit, 9\spadesuit\}$

$n(N) = 4$

$n(R \cup N) = 10$

ex 2 when $\boxed{\text{sets overlap}}$

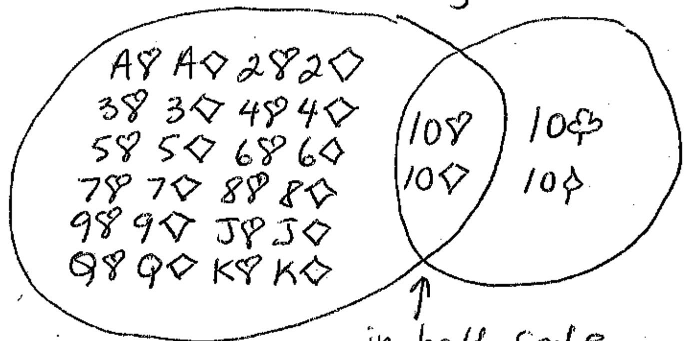
Draw a red card or a 10
(R) (T)

$R = \{A, 2, 3, \ldots K \text{ in hearts \& diamonds}\}$

$n(R) = 26$

$T = \{10♥, 10♦, 10♣, 10♠\}$

$n(T) = 4$



in both sets so subtract

$n(R \cap T) = 2$

$*\ \cap$ overlap

so.... $n(R \cup T) = n(R) + n(T) - n(R \cap T)$

$= 26 + 4 - 2$

$= 28$

$\boxed{4.2}$

factorial notation - $n!$ ←

ex $3! = 3 \times 2 \times 1$

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

so $n! = n(n-1)(n-2)(n-3) \ldots 1$

permutation - # of possibilities

* order matters; ab is different than ba

evaluate: * use brackets for fractions

Calculator $\frac{10!}{2!\,3!} \Rightarrow (10!) \div (2! \times 3!)$

Simplify

$$\frac{12!}{9!\,3!} = \frac{12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1 \times 9\times8\times7\times6\times5\times4\times3\times2\times1}$$

= cross off sam on top + bottom

$$= \frac{12\times11\times10}{3\times2\times1} = \frac{1320}{6} = 220$$

using n

$$\frac{(n+1)!}{(n-1)!} = \frac{(n+1)(n)(n-1)(n-2)(n-3)\ldots}{(n-1)(n-2)(n-3)\ldots}$$

= cross off same on top + bottom

$$= (n+1)(n)$$
$$= n^2+n$$

Solving

* Simplify 1st $\frac{n!}{(n-2)!} = 90$

* factorial notation only allowed to be positive

$$\frac{(n)(n-1)\cancel{(n-2)}\ldots}{\cancel{(n-2)}\ldots} \Rightarrow n(n-1) = 90$$

$$n^2 - n = 90$$

* move to one side + factor

$$n^2 - n - 90 = 0$$
$$(n+9)(n-10) = 0 \quad n = \cancel{-9} \text{ or } +10$$

4.3 Permulations

${}_rP_n$ ⇒ Permutations to use on Calculator

$0! = 1$

r = total number of things
n = number chosen.

$${}_rP_n = \frac{n!}{(n-r)!}$$

ex choose 6 songs to create a playlist out of 40 songs

$${}_{10}P_6 = \frac{10!}{(10-6)!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5$$
$$= 151200$$ different playlists

ex 2 how many passwords can be made

(A)
- no repeats are allowed
- you can use any number (0-9)
- any letter (upper or lower case) and #, !, $, * or ?
- the password must be 7 characters

(1) how many possibilities

a) 10 numbers (0-9)
b) 26 letters x 2 (upper + lower)
c) 5 symbols

67 choices

(2) ${}_{67}P_7 = 4383026968000$ different passwords.

(B) Same as A info but you can choose a password length from 6 characters to 10 characters

6 characters $_{67}P_6$ = 7185290120

7 characters $_{67}P_7$ = 438302696800

8 characters $_{67}P_8$ = 2629816181000000

9 characters $_{67}P_9$ = 155159154700000000

10 characters $_{67}P_{10}$ = 8999230971000000000

* <u>add</u> = 9157064491000000000 different passwords.

<u>note</u> → <u>Add</u> when seperate events
<u>multiply</u> when one event ~~a~~affects the other or the total end "view"

Conditional Permutations:

<u>ex</u> there are 7 people going to a movie
3 are adults
4 are children
The adults will sit in the middle and both ends.

<u>How many ways can they sit:</u>

multiply because overall arrangement is effected by each group.

<u>adults</u>	<u>children</u>
⊗ o o ⊗ o o ⊗	o ⊗ ⊗ o ⊗ ⊗ o
3 spots, 3 adults ⇒ $_3P_3$	4 children, 4 spots ⇒ $_4P_4$

$\Rightarrow {}_3P_3 \cdot {}_4P_4$
= 144 ways

$\boxed{4.4}$ Permutations – $\boxed{\text{Identical Objects}}$

things set { a a b c c b b b c a c c }

3 a's
4 b's > total 12
5 c's

$$P = \frac{n!}{a!\,b!\,c!} = \frac{12!}{3!\,4!\,5!} = 12! \div (3! \times 4! \times 5!)$$

\# same →

$$= 27,720$$

letters *ex* How many ways can you re-arrange

MISSISSIPPI – when a P has to be on both ends

i) total letters 11
ii) P at both ends – so 9 left
iii) there are 4 – S, 4 – I and 1 – M

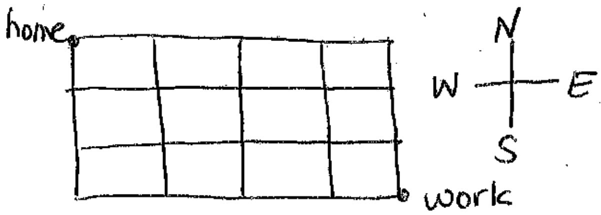
iv) $P \times \frac{9!}{4!\,4!} \times P = 1 \times 630 \times 1$

front

$= 630$ ways

NOTE M doesn't repeat so not on the bottom

direction *ex 2*



How many ways can you get to work (No back tracking)

4 → ways East
3 ↓ ways South

Total = 7 blocks!

$$\Rightarrow \frac{7!}{4!\,3!} = 7! \div (4! \times 3!)$$

$$= 35 \text{ ways}$$

$\boxed{4.5}$ Combinations (* order doesn't matter → ba is the same as ab)

Permutations of MATH - 2 letters VS **Combination of MATH - 2 letters**

Permutations — list

MA	AM
MT	TM
MH	HM
AT	TA
AH	HA
TH	HT

Combinations — list

MA	AT
MT	AH
MH	TH

$\boxed{4.6}$ Permutations VS Combinations

Permutations

- 1st, 2nd, 3rd place
- captain, co-chair
- president, VP, secretary
- books put on shelf
- photos in a book
- play list

Combinations

- people / objects chosen but not placed in order
- team (all)
- books to bring
- toppings
- songs to buy
- hands in cards

$\boxed{4.6}$ Combinations

- with conditions + without

without

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

n = total
r = # picked

ex choose 9 person team out of 20 people

$${}_{20}C_9 = \frac{20!}{9!(20-9)!} = 167960$$

with conditions

Combinations with conditions

→ break up into seperate events
& then multiply then add results

ex choose committee from
3 parents
6 teachers & 12 students

* has 5 members
& must have at least 1 teacher

At least means - could have 1, 2, 3, 4 or 5

1 teacher = ${}_6C_1 \cdot {}_{15}C_4$ = 8190

teacher others

2 teachers = ${}_6C_2 \cdot {}_{15}C_3$ = 6825

3 teachers = ${}_6C_3 \cdot {}_{15}C_2$ = 2100

4 teachers = ${}_6C_4 \cdot {}_{15}C_1$ = 225

5 teachers = ${}_6C_5 \cdot {}_{15}C_0$ = $6 \cdot 1 = 6$

total: 17,346 ways

$\boxed{4.7}$ Counting Problems

Permutation problem with conditions
* order matters

Group photo - 1 teacher, 3 boys + 5 girls.

A) Boys sit together + girls sit together.

□ □ □   teacher   □ □ □ □ □

3!   1   5!

B) How can they be arranged?

BTG or TBG or BGT
GTB   TGB   GBT } 6 ways or 3!

So → $\underbrace{(3! \times 1 \times 5!)}_{(A)} \times \underbrace{3!}_{(B)} = 4320$ ways

Combinations at most

How many different 5-card hands can be dealt with at most 1 black:

At most means → could be 1 or 0

(A) 1 black card → $\underset{\text{Black}}{{}_{26}C_1} \cdot \underset{\text{red}}{{}_{26}C_4} = 26 \cdot 14950$

(B) 0 black cards → ${}_{26}C_0 \cdot {}_{26}C_5 = 1 \cdot 65780$

(seperate events) add   454,480