Foundations 12 - Class notes

Chapter 5

5.1 Exploring Probability:

fair game: when a game is equally likely to be won by either player

experimental probability: $\frac{\#\text{times event occurs}}{\text{total times}}$

theoretical probability: $\frac{\#\text{time possible to occurs}}{\text{total chances}}$.

Is a game fair?

→ 2 containers each with 1, 2, 3
i) person 1 wins if sum is greater than product
ii) person 2 wins if product is greater than sum

*how to check? → write out the possible results

	Sum	Product
1, 1	2 *	1
1, 2	2	3 *
1, 3	4 *	3
2, 2	4	4 ← same # of wins
2, 1	3 *	2
2, 3	5	6 6*
3, 1	4 *	3 3
1, 3	4 *	3
3, 2	5	6 *
	↑	↑
	* 5 wins	* 3 wins

This game is unfair

player 1 will win more.

5.2 Probability + Odds

A' ← will not (un-favourable outcomes)

Odds: in favour

ratio ⇒ will occur : will not occur
against P(A) : P(A')
⇒ will not occur : will occur
P(A') : P(A)

ex 1 what are odds in favour of choosing a face card.
→ 4 suits + 3 face cards = 12
total in deck = 52

Odds in favour = $\frac{12}{40}$ (face cards → 12, not face cards ← 40)

or 3 : 10

ex 2 odds from probability

remember probability = $\frac{\#\text{possibilities}}{\text{total}}$

if probability is $\frac{6}{15}$ then odds for
6 : (15-6)
= 6 : 9 or **2:3**
odds against
(15-6) : 6
= 9 : 6 or **3:2**

ex 3 Probability from odds:
→ go backwards

ODDS for = 11 : 6

then probability = $\frac{11}{(11+6)} = \frac{11}{17}$ or 65%

5.3 Probability using Counting Methods

① find number of outcomes possible
② find total outcomes

ex Chances of 3 boys winning a race out of 10 and coming in 1st, 2nd, or 3rd.
* order matters so use permutations

① 3 places & 3 boys ⇒ ${}_3P_3 = 6$ ways

② 10 boys total, want 3 ⇒ ${}_{10}P_3 = 720$

$P = \frac{①}{②} = \frac{6}{720}$ or 0.83%

ex2 Probability that at least one out of 5 children is a boy.

① find probability that all 5 children are girls
⇒ 2 choices (either boy or girl)

So 5 children ⇒ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ ways

all girls ⇒ $1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

So probability that all are girls $\frac{1}{32} = 3.13\%$

∴ So probability that there is at least 1 boy
= 100% − (prob all girls)
= 100% − 3.13% = 96.9%

5.4 Mutually Exclusive Events

Mutually exclusive means – sets do not overlap

ex. $A = \{A, B, C, D\}$

$B = \{1, 2, 3, 4\}$

not mutually exclusive:

ex. $A = \{\text{multiples of } 2\} = \{2, 4, \underline{6}, 8, 10, \underline{12}, 14, 16, \underline{18} \ldots\}$

$B = \{\text{multiples of } 3\} = \{3, \underline{6}, 9, \underline{12}, 15, \underline{18}, 21 \ldots\}$

Determine the probability of rolling a sum greater than 8 or a multiple of 5.

① make sum chart ~~36~~ ← $(6 \times 6 = 36)$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

B

A

A = greater than 8

B = multiple of 5

$P(A) = \frac{10}{36}$

$P(B) = \frac{7}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

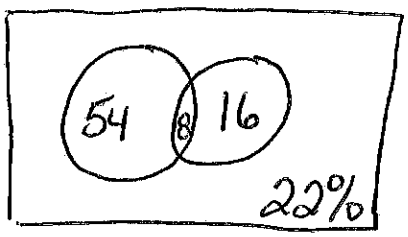
$= \frac{10}{36} + \frac{7}{36} - \frac{3}{36}$

$= \frac{14}{36}$ or 39%

5.4 Find Probability that a randomly selected person skips breakfast and not lunch

62% skip breakfast 22% eat both
24% skip lunch

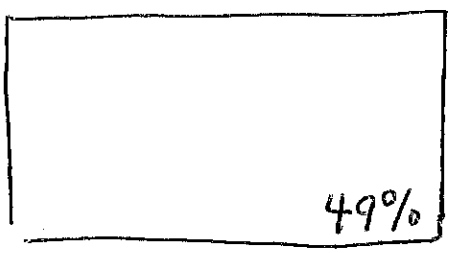
62+24=108%
so % skip both



* so 54% - skip breakfast & not lunch.

Find probability that a car has both a sunroof & heated seats

heated seats - 43%
neither - 49%
Sunroof - 36%



Since 49% have neither, 51% (100-49) must have either heated seats or a sunroof.
(H) (S)

So

* overlap has both!

$$P(H \cup S) = P(H) + P(S) - P(H \cap S)$$

$$51\% = 43\% + 36\% - P(H \cap S)$$

$$51\% = 79\% - \square \quad \square = 28\%$$

So 28% hath both

5.5 Conditional Probability

independent events - not affected by previous events

dependent events - affected by other events

conditional probability: probability that an event will occur after another events has occured

Ex 100 chips - 3 will be defective

Draw 2 chips - probability both defective?

① 1st chip - $\frac{3}{100}$; 2nd chip - ?

② If the first chip is defective then $\frac{2}{99}$

If the first chip is not defective → $\frac{3}{99}$

However, we want probability of both defective so...

A → 1st defective B - 2nd defective

P(B|A) - means 1st defective then 2nd defective

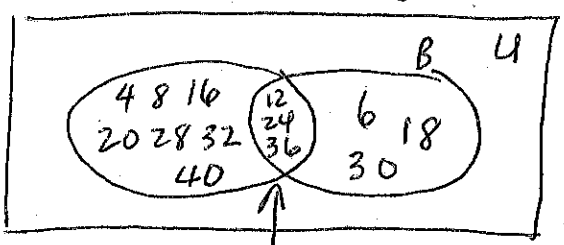
So $P(A \text{ and } B) = P(A) \cdot P(B|A)$

$= \frac{3}{100} \cdot \frac{2}{99} = \frac{6}{9900}$ or %

Probability of a pair of dependent events

Choose a number between 1 and 40.
The number is a multiple of 4.
What is the probability it is also a mulipte of 6?

mult of 4 = A = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40}
mult of 6 = B = {6, 12, 18, 24, 30, 36}
#1-40 = U = {1, 2 ... 40}



3 #'s are mult of 4 & mult of 6

5.5 cont

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

or $P(A \cap B) = P(A) \cdot P(B/A)$

$P(B/A)$ = probability of B happening after A

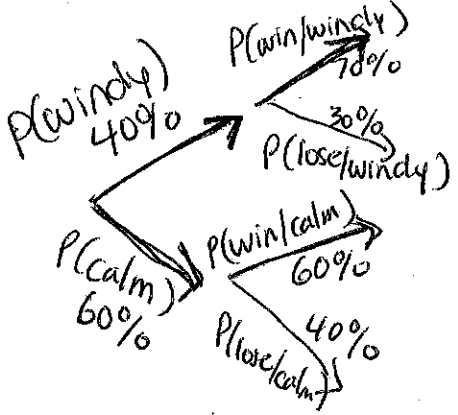
$P(A)$ = probability of A

$P(A \cap B)$ = probability of both

ex $P(\text{win})$ 60% of win on calm day
70% of win on windy day

* there is a 40% chance of high winds

$P(\text{win}) = P(\text{windy} \cap \text{win}) + P(\text{calm} \cap \text{win})$



$\rightarrow$ 40% * 70% = 28%

$\rightarrow$ 60% * 60% = 36%

$\therefore$ $P(\text{win}) = 28 + 36 = 64\%$

5.6 Independent Events

$$P(A \cap B) = P(A) \cdot P(B)$$

ex Probability of rolling a 4 and flipping heads

$$P(4) = \frac{1}{6} \quad \leftarrow 1 \text{ is a } 4 \quad \leftarrow 6 \text{ choices}$$

$$P(H) = \frac{1}{2} \quad \leftarrow 1 \text{ is heads} \quad \leftarrow 2 \text{ choices}$$

$$P(H/4) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

ex 2 Probability of choosing a red card under 10 then not rolling a 2.

under 10
2, 3, 4, 5, 6, 7, 8, 9
$= 8 \times 2$
♥ + ♢

$$P(\text{Red under } 10) = \frac{16}{52} \leftarrow \text{total cards}$$

* reduce $\frac{4}{13}$

$$P(\text{not a } 2) = \frac{5}{6} \quad \leftarrow 5 \text{ are not 2} \quad \leftarrow 6 \text{ choices}$$

$$P(2/R) = \frac{4}{13} \times \frac{5}{6} = \frac{20}{78} = \frac{10}{39}$$