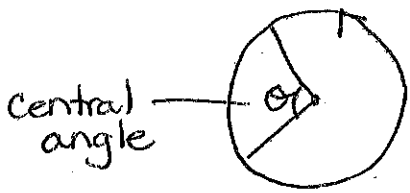
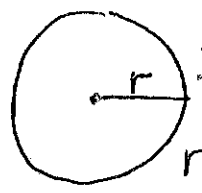
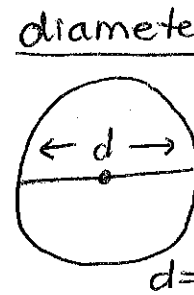
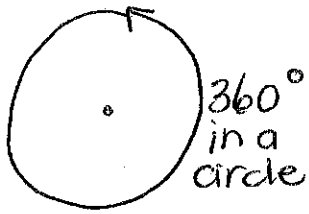
Math 12F – Classnotes
Chapter 8

8.1 Understanding Angles:



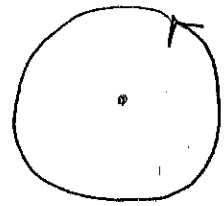


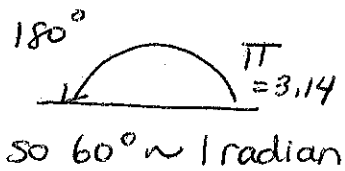




Radians

$360° = 2\pi = 6.28$





so $60° \sim 1$ radian

degree → radian

$$\frac{x° \cdot \pi}{180} = \text{radians}$$

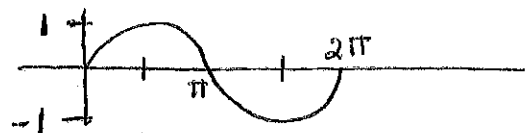
radian → degree

$$\frac{\text{radians} \cdot 180}{\pi} = x°$$

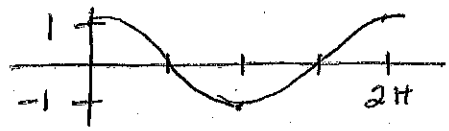
8.2 Periodic Functions

Domain $x \in \mathbb{R}$ | amp: $a = 1$
Range $-1 \leq y \leq 1$ | period: 2π
midline $y = 0$

$y = \sin x$



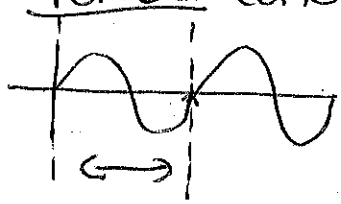
$y = \cos x$



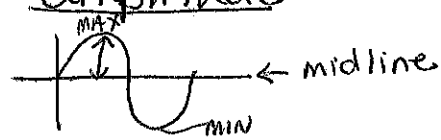
remember

* $\pi = 180°$ $\quad 2\pi = 360°$
$\frac{\pi}{2} = 90°$ $\quad \frac{3\pi}{2} = 270°$

Period (one cycle)



amplitude

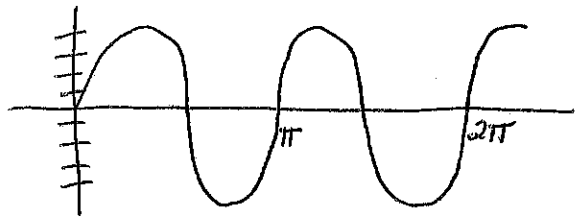


* how high from centre line (midline)

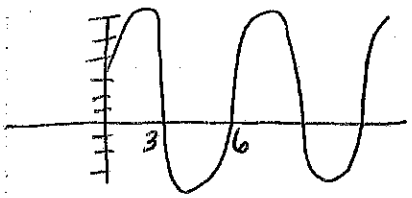
8.3 Graphs of Sinosodial Functions

Period → length of one cycle
Amplitude → height from midline
Range { maximum → highest y value
minimum → lowest y value
midline → $\frac{\text{maximum} + \text{minimum}}{2}$

ex

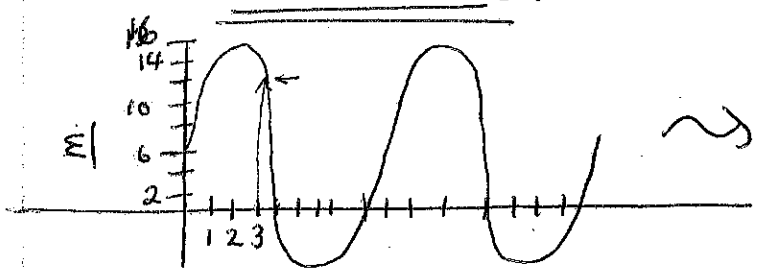


amplitude = 4 max = 4, min = −4 midline ⇒ y = 0
period = π



max = 6 min = −3
midline = $\frac{6 + -3}{2} = \frac{3}{2}$ or 1.5
amplitude = 6 − 1.5 = 4.5
period = 6

What is the height of the ferris wheel at 3 seconds?



≈ about 14m

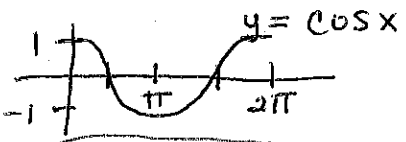
8.4 Equations of Sinusoidal Functions

$$y = a \sin b(x - c) + d$$

a = amplitude
b = period $\frac{2\pi}{b}$

c = phase shift (+left, −right)
d = equation of the midline

ex $y = 2\cos 4x + 1$

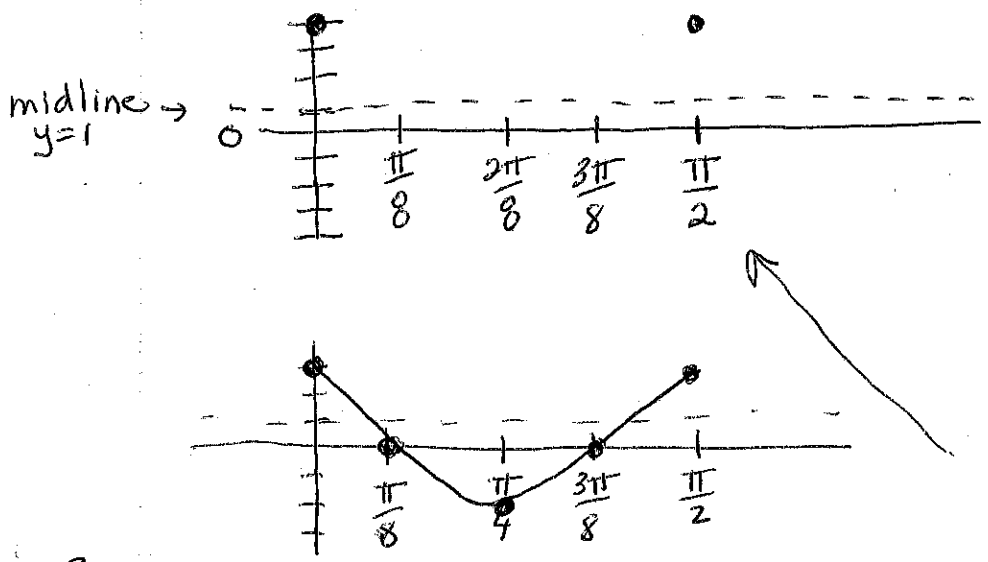


amplitude = 2
midline ⇒ y = 1
period = $\frac{2\pi}{4} = \frac{\pi}{2}$

orig graph pattern
- at 0 y = 1
- at π y = −1
- at 2π y = 1
½ way ($\frac{\pi}{2} + \frac{3\pi}{2}$) - y = 0

Draw it out:

period = $\frac{\pi}{2}$ ⇒ so at 0 + $\frac{\pi}{2}$ → y = highest
+ amplitude = 2 and midline - y = 1
so highest pt is at y = 3



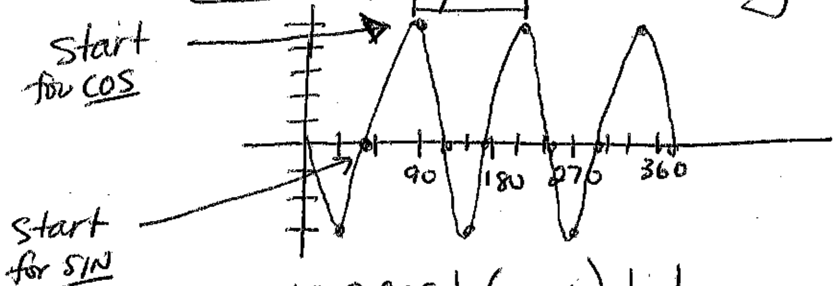
① put in midline
② put in scale
③ place high pts $(0, \frac{\pi}{2})$ at 3
④ redo scale to find ½ way + 2 other points between
ex ½ of $\frac{\pi}{2}$ - is $\frac{\pi}{4}$
½ of $\frac{\pi}{4}$ is $\frac{\pi}{8}$
∴ new scale $\frac{\pi}{8}$
⑤ fill in highs, lows + zeros + connect dots

Graphing calc

GRAPH put in equation DRAW V-Window TRIG → G SOLV
*use brackets if inequation

Radians?
shift SETUP MENU
Angle → V RAD
EXE

Match equation to graph



$y = a\cos b(x-c) + d$

(a: ampl; b: period; c: l or r; d: midline)

amplitude $= \dfrac{max - min}{2}$

$= \dfrac{5-(-3)}{2} = 4$

midline = max − amplitude

$= 5 - 4 = 1$

phase shift → right 90

period $= \dfrac{2\pi}{b}$ or $\dfrac{360}{b}$ ←

90 → 210
= 120 →

$= \dfrac{360}{120} = 3$

equation

Cos

$y = 4\cos 3(x - 90) + 1$

(4: amplitude; 3: period; 90: phase shift; 1: midline)

SIN

Could also be a sin graph: all depends on where you start

$y = a\sin b(x-c) + d$

midline = 1
amplitude = 4
change * phase shift = 60
period = 3

$y = 4\sin 3(x - 60) + 1$

---

amplitude: $\dfrac{max - min}{2}$

midline: max − amplitude

period: $\dfrac{2\pi}{b}$ or $\dfrac{360}{b}$ or from graph $\dfrac{2\pi}{period}$ or $\dfrac{360}{period} = b$

phase shift: distance
SIN: 1st y=0 to next y=0
COS: 1st max to 2nd max