CHAPTER 2: MATH 12PC NOTES

2.1 (D) Domain – x values allowed
(R) Range – y values allowed

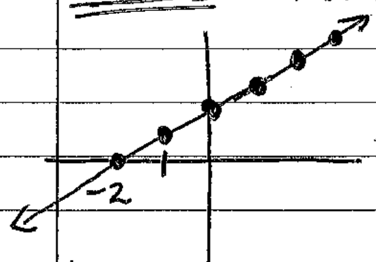
$\boxed{\text{* look under } \& \text{ in}}$

$\sqrt{\ }$ cannot be negative so $x \geq 0$ $\therefore$ $y \geq 0$

$\frac{1}{x}$ ← denominator cannot be zero

To sketch $\sqrt{f(x)}$ use original graph $(f(x))$ & $\sqrt{y}$

linear $f(x)$



$\boxed{\begin{array}{l} D - x \in R \\ R - y \in R \end{array}}$

NO LIMITS

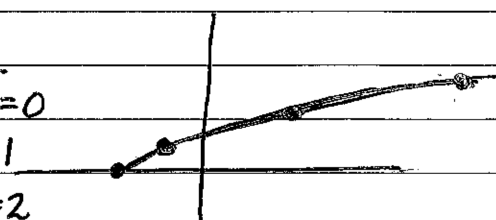
x	y	
-2	0	*
-1	1	*
0	2	
1	3	
2	4	*
3	5	
4	6	
5	7	
6	8	
7	9	*

* choose values you can $\sqrt{\ }$ evenly

$\Rightarrow$

x	y
-2	$\sqrt{0}=0$
-1	$\sqrt{1}=1$
2	$\sqrt{4}=2$
7	$\sqrt{9}=3$

$\sqrt{f(x)}$ radical



$\boxed{\begin{array}{l} D \ x \geq -2 \\ R \ y \geq 0 \end{array}}$

* limits → $\sqrt{\ }$ can't be neg.
$\therefore$ stops at ~~x=~~ -2
this makes $y \geq 0$

2.2 Graphing Rational Functions:

i) Vertical asymptotes: when x value makes denominator = 0

ex $\frac{1}{x} \Rightarrow$ asymptote $x = 0$

$\frac{1}{x-2} \Rightarrow$ asymptote $x = 2$

$\frac{1}{(x)(x+1)(x-1)} \Rightarrow$ asymptotes at $x = 0, -1, +1$

ii) **Holes** → a vertical asymptote changes to a hole when it can be crossed off when simplifying

* remember to note both x & y values of hole when stating Domain & Range.

ex 1: $y = \frac{x^2 - 1}{x + 1}$ * $x \neq -1$

FACTOR FIRST

Simplify

$$y = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} = x - 1$$

still cannot be $= -1$ even though there is no longer a denominator because in original equation it was there.

Domain: $x \neq -1$

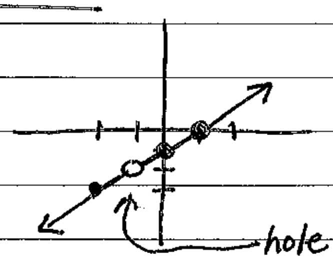
Range → Since $x \neq -1$

to find limits on range → plug into simplified equation $y = x - 1$

$y = (-1) - 1 = -2$

So... $y \neq -2$

To graph use the simplified equation



$y = x - 1$

x	y
-2	-3
0	-1
1	0

* dont forget

$\left.\begin{array}{l} x \neq -1 \\ y \neq -2 \end{array}\right\}$ hole

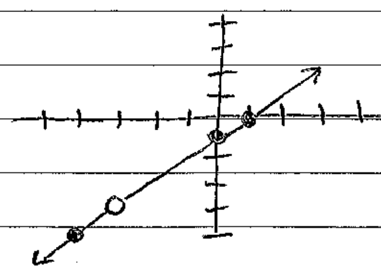
ex 2 $y = \frac{x^2 + 2x - 3}{x + 3} = \frac{\cancel{(x+3)}(x-1)}{\cancel{x+3}} = x - 1$ ②

① D: $x \neq -3$

③ R ⇒ $(-3) - 1$

$y \neq -4$

x	y
-4	-5
0	-1
1	0

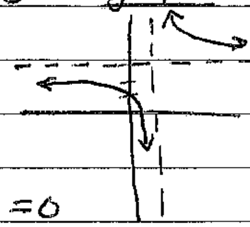


iii) horizontal asymptotes

ex 1 $\boxed{\dfrac{3x}{x-1}}$ (**same** degree of x on top & bottom)
-cannot factor
vertical asymptote $x=1$

horizontal asymptote $\Rightarrow$ since $x \neq 1$

$\dfrac{3x}{x-1} = \dfrac{3(1)}{(1)-1}$ ← y will never be 3 graph

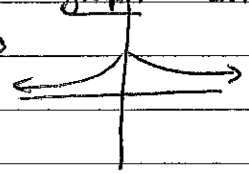


Rule #1: same degree ÷ co-efficients.

Domain: $x \neq 1$ Range: $y \neq 3$

ex 2 $\boxed{\dfrac{6}{x^2+2}}$ ← no variable on top so can never =0
∴ horizontal asymptote $y=0$
cant make denominator =0 ∴ no vertical asymptote

D: $x \in R$ graph -also cant make y=negative
R: $y>0$ ∴ $y>0$



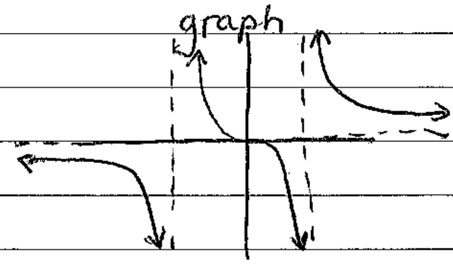
Rule #2:
degree higher by 1 in denominator
horiz. asymptote $y=0$

ex 3 $\dfrac{x}{x^2-4}$ ← degree higher by 1 in denominator
horizontal asymptote $y=0$

Factor to find vertical asymptotes $\dfrac{x}{(x-2)(x+2)}$ $x \neq 2, -2$

D: $x \neq 2, -2$
R: $y \neq 0$

graph



2.3

iv) **Oblique asymptote**: (degree higher on top) (this is why we learned long division! → remember a fraction is a division statement)

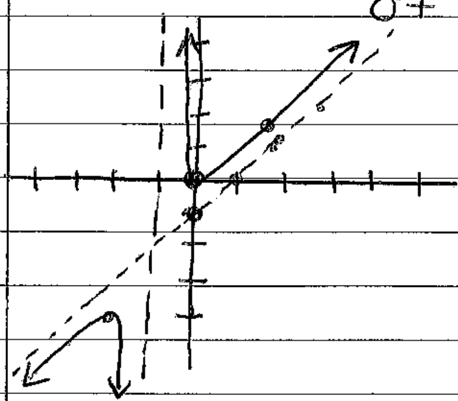
ex 1

$$y = \frac{x^2}{x+1} \leftarrow x \neq -1 \text{ vert asymp}$$

To find asymptotes do long division

$$
\begin{array}{r l}
 & x - 1 \quad \leftarrow \text{equation of oblique asymptote.} \\
x+1 \,\big)\!\! & \overline{x^2 + 0x + 0} \\
 & -\,x^2 + x \\
\hline
 & \quad -x + 0 \\
 & -\;\, -x - 1 \\
\hline
 & \qquad\quad 1 \quad \leftarrow \text{too small to worry about}
\end{array}
$$

<u>To graph</u> → put in asymptotes + do a table of values on all sides of asymptotes.



x	y
0	0
1	$\frac{1}{2}$
2	$\frac{4}{3}$
-2	-4

D $x \neq -1$

R $y \neq x - 1$

ex 2 $y = \dfrac{x^2 + x - 2}{x+1}$

Factor 1st ←

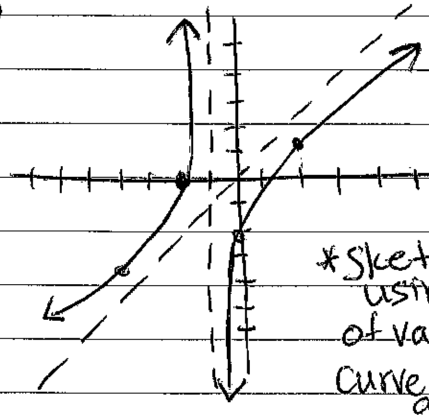
$\dfrac{(x+2)(x-1)}{x+1}$

* can't reduce

① $x \neq -1$

②
$$
\begin{array}{r l}
 & x + 0 \\
x+1 \,\big)\!\! & \overline{x^2 + x - 2} \\
 & -\,x^2 + x \quad \downarrow \\
\hline
 & \qquad 0x - 2
\end{array}
$$

oblique asymptote $y = x$



*Sketch using table of values + curve around asymptotes

③

x	y
0	
2	
-2	
-4	

2.4 Sketching Graphs of Rational Functions

To sketch: FACTOR FIRST then find the following:

1. vertical asymptotes
2. holes
3. horizontal asymptotes
4. oblique asymptotes
5. simplified version of the function.
6. x intercepts ($y=0$) y intercepts ($x=0$)

ex 1 $\boxed{\dfrac{x-1}{x^2-1}}$ $\quad \dfrac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \dfrac{1}{(x+1)}$ ← vertical asymptote at $x=-1$

hole at $x=1 \rightarrow \frac{1}{(1+1)} = \frac{1}{2}$

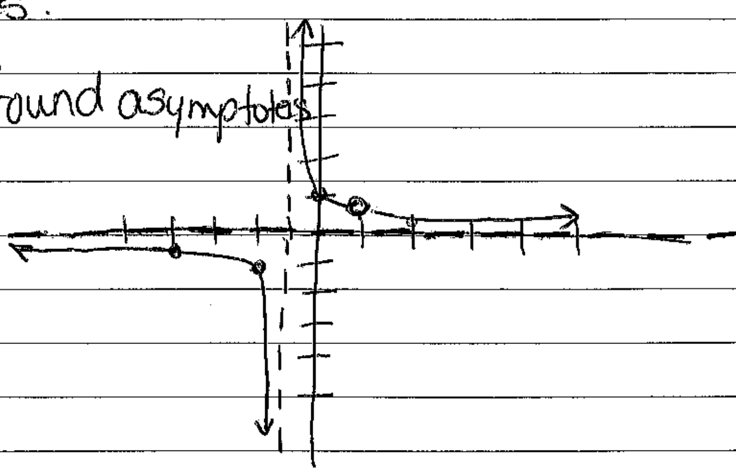
hole $(1, \frac{1}{2})$

horizontal asymptote: since degree is 1 more in denominator
$y=0$

To graph
a) sketch in all asymptotes &
b) do a table of values on both sides of asymptotes.
c) add in holes
d) connect & curve around asymptotes

x	y
-4	$-\frac{1}{3}$
-2	-1
0	1 (y intercept)
2	$\frac{1}{3}$

hole at $(1, \frac{1}{2})$



ex 2 $\boxed{\dfrac{2x^2+5x-12}{x+2}} \Rightarrow \dfrac{(x+4)(2x-3)}{x+2}$

* cant simplify so no hole

* degree is larger in numerator $\therefore$ oblique asymptote

vertical asymptote $x=-2$

$$
\begin{array}{r}
2x+1 \\
x+2 \;\big)\overline{2x^2+5x-12} \\
\underline{-2x^2+4x} \downarrow \\
x-12 \\
\underline{-\;x+2} \\
-14
\end{array}
$$

oblique $\Rightarrow$ $y=2x+1$

xintercepts (what would make numerator $=0$)

$\dfrac{(x+4)(2x-3)}{x+2}$ $x=-4, \frac{3}{2}$

yintercepts (make $x=0$)

① draw all asymptotes
② plot x & y intercepts
③ curve around asymptotes & connect dots.