Math 12PC - Chapter 7 notes

$\boxed{7.1}$ To solve Trig Equations graphically

Use your graphing calculator

ex. $2\cos x = 0$ from $0 \leq x < 2\pi$

(1) $y = 2\cos x$
$y = 0$

(2) set window
Xmin=0
Xmax=2π

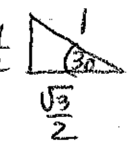
(3) press graph.

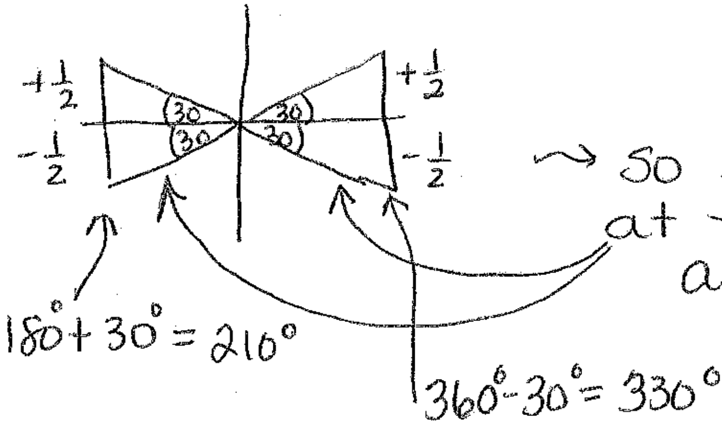
(4) use trace to find intersection
or go to CALC and use intersection (5)
(2nd Trace)

$\boxed{7.2}$ Solve Trig Equations Algebraically

Remember the unit circle.

$\sin\theta = -\frac{1}{2}$ between $0°$ and $360°$

Ask - where is $\sin\theta = -\frac{1}{2}$ ~ $\sin\theta = \frac{1}{2}$ at $30°$ 

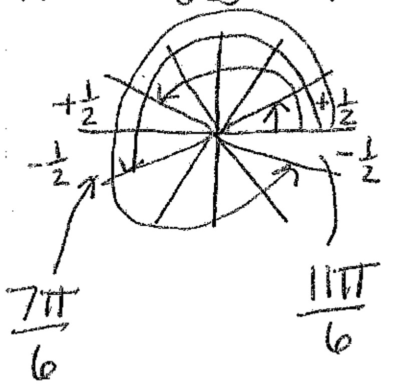


~ so $\sin\theta$ is $-\frac{1}{2}$ at these 2 angles

So ... $\sin\theta = -\frac{1}{2}$ at $210°$ and $330°$

now use radians. $\sin\theta = \frac{1}{2}$ at $30°$

or at $\frac{180}{30} = \frac{\pi}{6}$



* Count # of divisions to get radians.

so $\sin\theta = -\frac{1}{2}$ at $\frac{7\pi}{6}$ & $\frac{11\pi}{6}$

To find the general solution use above at add period x (k)

$\therefore$ general solution for $\sin\theta = -\frac{1}{2}$ → period $= 2\pi$

is $\frac{7\pi}{6} + 2\pi k$ and $\frac{11\pi}{6} + 2\pi k$

Using algebra to rearrange:

ex 1 $\quad 2\cos x + 1 = 0$

$\quad (-1) \; (-1)$

$\Rightarrow \dfrac{2\cos x}{(2)} = \dfrac{-1}{(2)}$

$\Rightarrow \cos x = \dfrac{-1}{2}$

ex 2 $\quad 7 + 2\sin x = 4\sin x + 5$

$\quad (-7) \qquad\qquad (-7)$

$2\sin x = 4\sin x - 2$

$(-4\sin x) \; (-4\sin x)$

$\dfrac{-2\sin x}{-2} = \dfrac{-2}{-2} \qquad \sin x = 1$

Ask - where is $\sin x = 1$ $\qquad \sin 90° = -1$

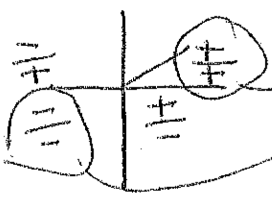
$\sin 270° = 1$

$\therefore \; x = 270°$ or $\dfrac{3\pi}{2}$

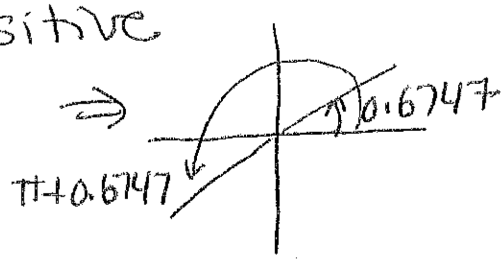
* when it doesnt work out to an exact value

$\tan x = \dfrac{4}{5} \rightsquigarrow$ find angle $\tan^{-1}\left(\frac{4}{5}\right) = 0.6747$ (in radians)

now look at quadrants - which ones would tan be positive



1 & 3 $\Rightarrow$



$\therefore$ Solution is 0.6747 and $\pi + 0.6747$ or 3.18163

7.8 $\boxed{4\sin^2 x = 3}$ $\qquad -360^\circ \le x \le 0^\circ$

$(\div 4) \quad (\div 4) \Rightarrow \sin^2 x = \frac{3}{4}$ * square root both sides.

$$\sin x = \pm\sqrt{\frac{3}{4}} \text{ or } \frac{\pm\sqrt{3}}{2}$$

To find the angle

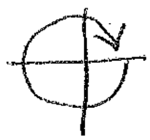
$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

now + or − : $-360^\circ + 60^\circ = -300^\circ$

$-360^\circ \le x \le 0^\circ$ $\qquad -180^\circ + 60^\circ = -120^\circ$

$-180^\circ - 60^\circ = -240^\circ$

$0^\circ - 60^\circ = -60^\circ$



Using the quadratic formula:

$3\cos^2 x + \cos x - 1 = 0 \qquad \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 3 \quad b = 1 \quad c = -1$

$$\frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)} = \frac{-1 \pm \sqrt{1+12}}{6} = \frac{-1 \pm \sqrt{13}}{6}$$

find angle $\cos^{-1}\left(\frac{-1+\sqrt{13}}{6}\right)$ and $\cos^{-1}\left(\frac{-1-\sqrt{13}}{6}\right)$

7.3 Reciprocal and Quotient Identities

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Also... using algebra to rearrange....

$\csc\theta\sin\theta = 1$ $\sec\theta\cos\theta = 1$ $\cot\theta\tan\theta = 1$

and $\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

and $\tan\theta\cos\theta = \sin\theta$; $\cos\theta = \frac{\sin\theta}{\tan\theta}$

$\cot\theta\sin\theta = \cos\theta$; $\sin\theta = \frac{\cos\theta}{\cot\theta}$

Prove and identity using known values

$$\frac{\sin\theta + \cos\theta}{\sin\theta} = 1 + \cot\theta \quad \text{use} = \frac{\pi}{4}$$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{4} = 1$ so $\cot\frac{\pi}{4} = 1$

$\dfrac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} =$	$1+1$
	$= 2$
$= \dfrac{\frac{2}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$	

$= \frac{2}{\sqrt{2}} \div \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{1}$

$= 2$

non-permissible values - denominator cannot $= 0$

$$\frac{\sin\theta + \tan\theta}{1+\cos\theta} = \sin\theta \sec\theta$$

$\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta \neq 0$

so

$\theta \neq \frac{\pi}{2}$

$\cos\theta$ cannot $= -1$

$\theta \neq \pi$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos\theta \neq 0$

so $\theta \neq \frac{\pi}{2}$

Solving without values:

$\frac{\sin\theta + \tan\theta}{1+\cos\theta}$	$= \sin\theta \sec\theta$
make denom. the same. $\frac{\sin\theta + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	$= \sin\theta \frac{1}{\cos\theta}$
	$= \frac{\sin\theta}{\cos\theta}$
$\frac{\frac{\cos\theta\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	
$\frac{\frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta}}{1+\cos\theta}$	
$= \frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta(1+\cos\theta)}$	

think like $\div$ by a fraction

$$\frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta} \div 1 + \cos\theta$$

$$= \frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta} \times \frac{1}{1+\cos\theta}$$

$$= \frac{\cos\theta\sin\theta + \sin\theta}{\cos\theta(1+\cos\theta)}$$

now Factor $\frac{\sin\theta\,\cancel{(\cos\theta + 1)}}{\cos\theta\,\cancel{(1+\cos\theta)}}$

$= \frac{\sin\theta}{\cos\theta}$

7.4 The Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

also $\sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta = \sec^2\theta - 1$ or $\sec^2\theta - \tan^2\theta = 1$

$\cot^2\theta = \csc^2\theta - 1$ or $\csc^2\theta - \cot^2\theta = 1$

ex. $\csc\theta\cos^2\theta + \sin\theta \stackrel{?}{=} \csc\theta$

Left side:

$\frac{1}{\sin\theta}\cos^2\theta + \sin\theta$

* make denominators equal $\quad \frac{\cos^2\theta}{\sin\theta} + \sin\theta$

$\frac{\cos^2\theta + \sin\theta\sin\theta}{\sin\theta}$

* pyth identity $=1$ $\quad \frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

Right side:

$= \frac{1}{\sin\theta}$

Solve by factoring & using identities.

$2\cos^2 x - 3\sin x = 0$

$2(1-\sin^2 x) - 3\sin x = 0$

$\div -1$ $\quad 2 - 2\sin^2 x - 3\sin x = 0$

to make positive so you can factor $\quad -2 + 2\sin^2 x + 3\sin x = 0$

$2\sin^2 x + 3\sin x - 2 = 0$ think $2x^2+3x-2$

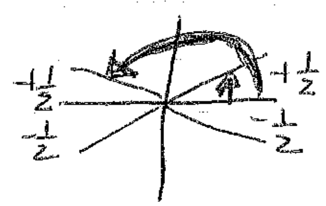
$(2\sin x - 1)(\sin x + 2) \leftarrow (2x-1)(x+2)$

So $\sin x = \frac{1}{2}$ or -2

What is the solution between π and 2π

$\sin x$ is not in values so -2 = no solution

but $\sin x$ can $= \frac{1}{2}$ at



So $x = \frac{5\pi}{6}$ and $\frac{\pi}{6}$

7.5 Sum and Difference Identities

$\sin(x+y) = \sin x \cos y + \cos x \sin y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

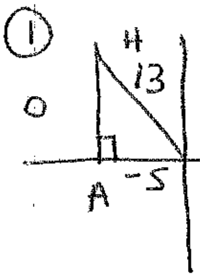
To use these identities:

(A) To find the difference between 2 angles in exact value: $\cos(x-y)$

given: (1) $\cos x = \frac{-5}{13}$ in Quadrant 2

$\sin y = \frac{3}{5}$ in Quadrant 1

(1)



$\cos x = \frac{-5}{13}$

to find $\sin x \Rightarrow$ use pyth. theorem

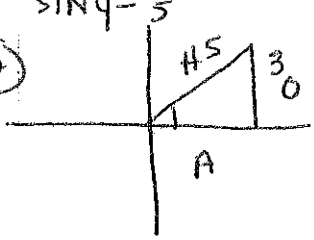
$\sqrt{13^2 - (-5)^2} = \sqrt{144} = \pm 12$

since in quad 2 O = positive

So $\sin x = \frac{12}{13}$

$\sin y = \frac{3}{5}$

(2)



to find $\cos y \Rightarrow$ use pyth theorem.

$\sqrt{5^2 - 3^2} = \sqrt{16} = \pm 4$

Since in quad 1 $\rightarrow$ A is positive

~~so cos fs~~ so $\cos y = \frac{4}{5}$

now use the formula:

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$= \left(\frac{-5}{13}\right)\left(\frac{4}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{16}{65}$

(B) To find the exact value of an angle:

$\sin 75°$ ⇝ $75°$ can be broken up into 2 known angles.

$45° + 30°$

So... $\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

(C) To Simplify:

$$\frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{12}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{12}} = \tan\left(\overset{\frac{2\pi}{12}}{\frac{\pi}{6}} + \frac{\pi}{12}\right)$$

$$= \tan\left(\frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{4}\right)$$

$$= 1$$

(D) To Prove an identity:

$\sin(\pi - x)$	$\sin x$
$\sin\pi\cos x - \cos\pi\sin x$	
$(0)\cos x - (-1)\sin x$	
$-(-\sin x)$	
$\sin x$	

(E) To solve:

$\cos 4x \cos x + \sin 4x \sin x = 1$

$\cos(4x - x) = 1$

$\cos 3x = 1$ $\quad\quad \cos\theta = 1 \rightarrow \theta = 0 \text{ and } 2\pi$

So $3x = 1$

↑ 3x more times as often

$\therefore x = \frac{0}{3}, \frac{2\pi}{3}$

7.6 Double angle identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

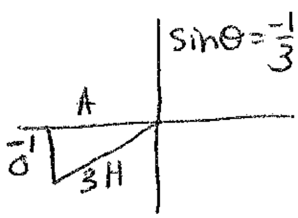
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

(A) Use to determine exact values

given $\sin\theta = -\frac{1}{3}$ in Quad. 3

find $\sin 2\theta$

① find $\cos\theta$



$\sqrt{3^2-(-1)^2} = \sqrt{8}$

* in quad 3 → so $-\sqrt{8}$ or $-2\sqrt{2}$

so $\cos\theta = \dfrac{(-2\sqrt{2})}{3}$

$$\sin 2\theta = 2\left(-\tfrac{1}{3}\right)\left(-\tfrac{2\sqrt{2}}{3}\right)$$
$$= \frac{4\sqrt{2}}{9}$$

B) To solve or prove identities!

ex1 $\cos^2\left(\frac{\pi}{4}\right) - \sin^2\left(\frac{\pi}{4}\right)$

$= \cos 2\left(\frac{\pi}{4}\right)$

$= \cos\left(\frac{2\pi}{4}\right) \Rightarrow \cos\frac{\pi}{2} \Rightarrow 0$

ex2 To solve - factor

$6\cos^2\theta - 3 \Rightarrow 3(2\cos^2\theta - 1)$

$= 3(\cos 2\theta)$

ex3 Use multiple identities.

$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$

$\csc 2\theta + 1 = \dfrac{(\sin\theta + \cos\theta)^2}{\sin 2\theta}$

look at all identities + rearrange-tip

$\dfrac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\sin 2\theta}$

$\dfrac{\underbrace{\sin^2\theta + \cos^2\theta}\ + \underbrace{2\sin\theta\cos\theta}}{\sin 2\theta}$

$\dfrac{\underbrace{1} + \underbrace{\sin 2\theta}}{\sin 2\theta} = \dfrac{1}{\sin 2\theta} + \dfrac{\sin 2\theta}{\sin 2\theta}$

$= \csc 2\theta + 1$

Trig identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

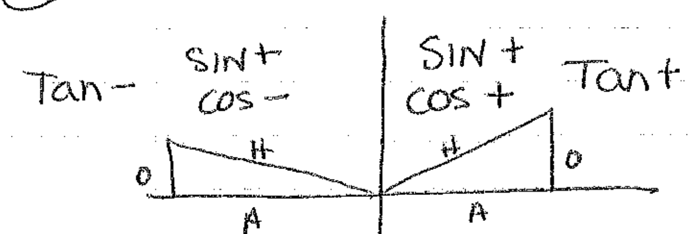
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$\sin 2\theta = 2\sin\theta\cos\theta$

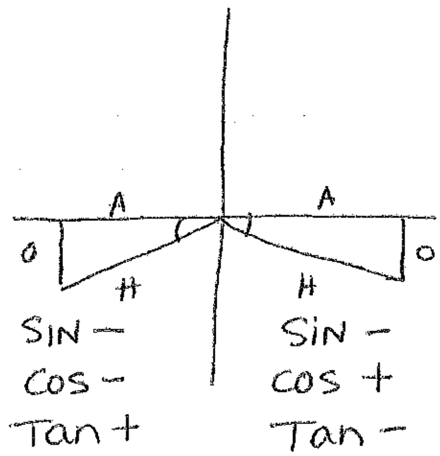
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

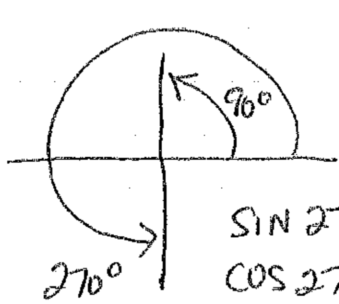
degrees	radians	SIN	COS	TAN
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undef.
180	π	0	-1	0



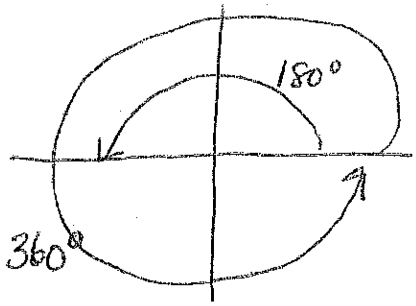
To find other angles use this 180 - angle.



SIN 90 = 1
Cos 90 = 0



SIN 270 = -1
COS 270 = 0



SIN 180 = 0
COS 180 = -1

SIN 360 = 0
Cos 360 = 1

SINCE $\text{Tan} = \frac{\text{SIN}}{\text{COS}}$ Tan ~ undefined at 90 & 270 → but tan = 0 at 180 + 360