Chapter 3 notes - MA12PC

3.1 Translating Graphs of Functions

$f(x)$ means the same as y

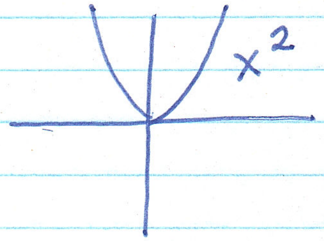
$$y = x + 2 \Leftrightarrow f(x) = x + 2.$$

Sometimes the letter infront changes but is just means a function

Recall from grade 11:

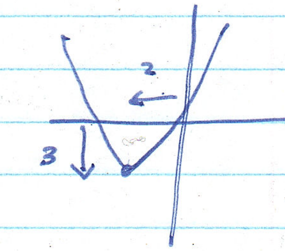
$$y = a(x-p)^2 + q$$

a: stretch / compression; p: left or right; q: up or down



$\Rightarrow$ $y = (x+2)^2 - 3$

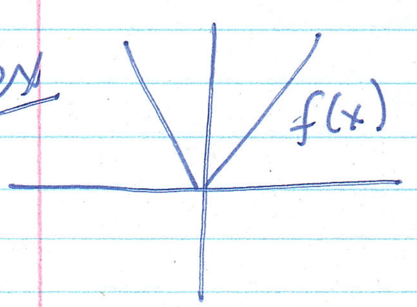
left 2, down 3



In grade 12 → you have to translate the graph 2 ways
- knowing the equation
- not knowing the equation and transforming points or $f(x)$

$y-k=f(x-h)$ is the same as $y=f(x-h)+k$

ex

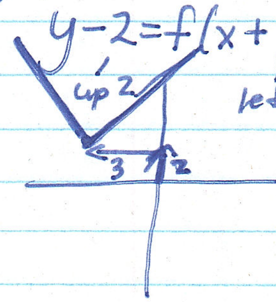


$$y - k = f(x - h)$$

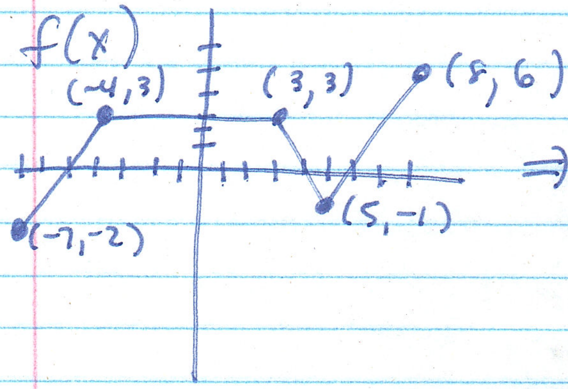
k: − up, + down; h: − right, + left

$y-2=f(x+3)$

up 2, left 3



To translate a complicated graph
- write down all the points where it changes direction; then translate



$\Rightarrow$

-7	-2
-4	3
3	3
5	-1
8	6

new function
translate 5 right and 2 down

equation:
$y+2 = f(x-5)$
5 right

new table of values
5 right = x + 5
2 down = y - 2

f(x)

-7	-2
-4	3
3	3
5	-1
8	6

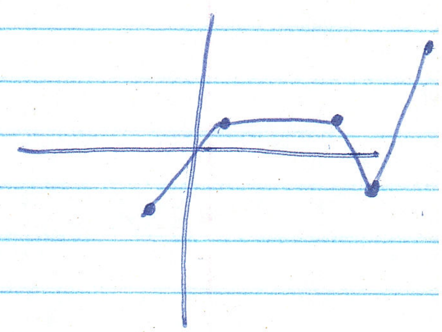
$\Rightarrow$ g(x) or y+2=f(x+5)

-7+5	-2-2
-4+5	3-2
3+5	3-2
5+5	-1-2
8+5	6-2

$\Rightarrow$

-2	-4
1	1
8	1
10	-3
13	4

new graph



remember

$y-k = f(x-h)$
- up / + down
- right / + left

If given equation - just add in values

$y = 2x+3 \Rightarrow$ translate 4 up and 3 right $y-4 = 2(x-3)+3$

$y = 2(x-3)+7$

3.2 Reflecting Graphs of Functions

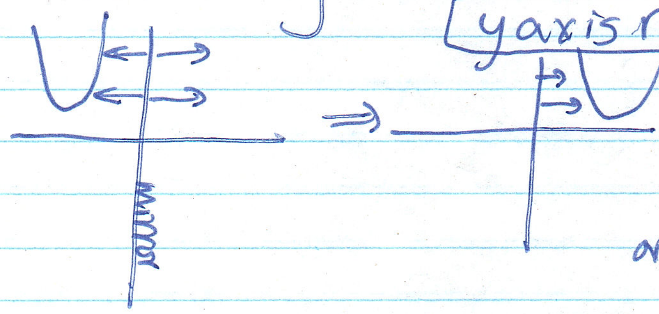
There are 3 ways to reflect graphs.

- → over y axis
- → over x axis

and over x=y line (covered in 3.5)

* think of a y axis reflection as holding a mirror on the y axis.

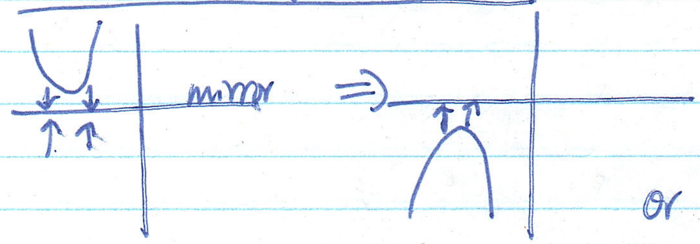
$\boxed{\text{y axis reflection}}$



* This changes
$f(x) \Rightarrow f(-x)$

or $2f(x+3) \Rightarrow 2f(-x+3)$

x axis reflection



This changes the
$f(x) \Rightarrow -f(x)$

or $2f(x+3) \Rightarrow -2f(x+3)$

Simple Rule

x reflection - outside
y reflection - inside

$\boxed{\text{orig graph}}$

-2	2
-3	6
-1	6

x reflection

- all y values change signs

x	y
-2	-2
-3	-6
-1	-6

y reflection

- all x values change signs

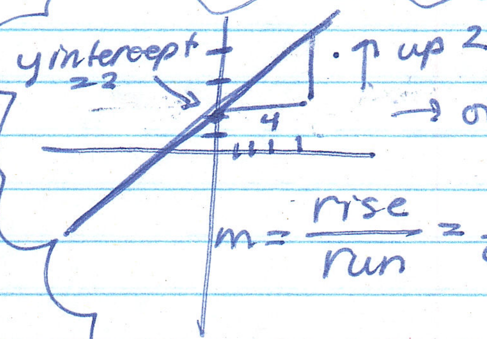
x	y
2	2
3	6
1	6

Grade 10 + 11 review

Writing the equation of a line:

$y = mx + b$ ← y intercept

m: slope or $\frac{\text{rise}}{\text{run}}$



$m = \frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$

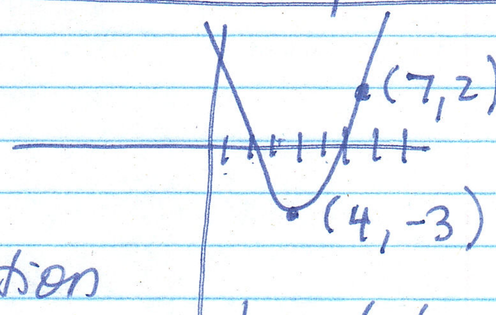
∴ your equation is

$y = \frac{1}{2}x + 2$

Writing the equation of a parabola:

$y = a(x-p)^2 + q$

(p, q): vertex



① plug vertex into equation

* dont forget to flip sign in brackets

you have $y = a(x-4)^2 - 3$ *use a point from graph to plug in + solve for $\underline{\underline{a}}$

② plug in point $(7, 2)$
(x, y)

$2 = a(7-4)^2 - 3$

$2 = a(3)^2 - 3$

$+3 \qquad +3$

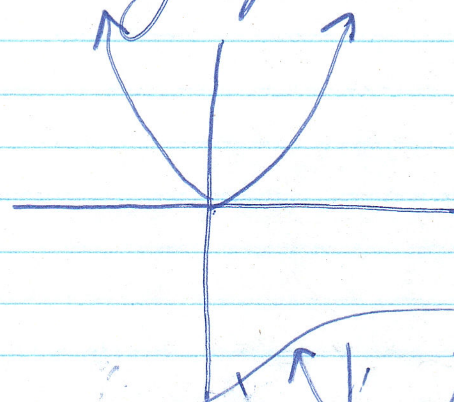
$5 = 9a$

$\frac{5}{9} = a$ ⇒ find equation ⇒ $y = \frac{5}{9}(x-4)^2 - 3$

Squared graphs

Standard graph

x	y
0	0
1	1
2	4
-2	4
3	9
-3	9

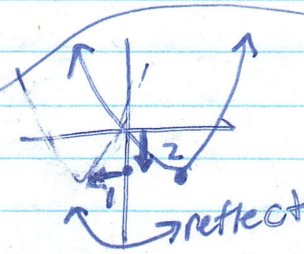


To translate start with this graph + translate points

$y+2=-(x+1)$

dom 2   reflect y   left 1

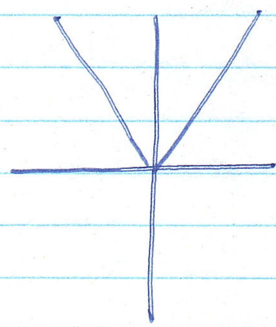
new graph



Absolute value graphs

$y=|x|$ standard graph

0	0
1	1
-1	1
2	2
-2	2

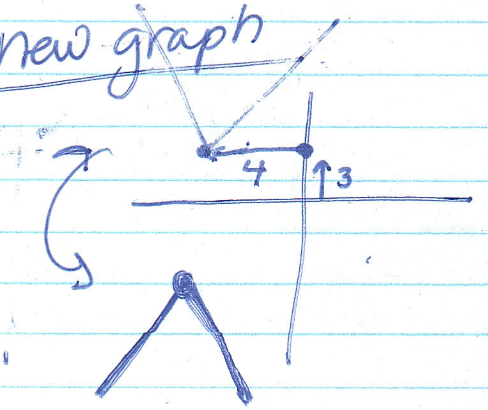


To translate start with this graph + translate points

ex. $y-3=-|x+4|$

up 3   reflect x   4 left

new graph



3.3 Stretching & Compressing Graphs

in a function → $f(x)$

Outside

$y = af(x)$

vertical stretch or compression

$|a| > 1 \Rightarrow$ vertical stretch

$|a| < 1 \Rightarrow$ vertical compression

*remember

$-af(x)$

means x reflection = treat it like 2 seperate transformations

ex $y = 2f(x) \Rightarrow$ vertical stretch by a factor of 2

$y = \frac{1}{3}f(x) \Rightarrow$ vertical compression by a factor of $\frac{1}{3}$

inside

in the function $y = f(x)$

$y = f(bx)$ means a horizontal stretch or compression

$|b| > 0$ - horizontal compression

$|b| < 0$ - horizontal stretch

** opposite of vertical

*remember $f(-bx)$ means y reflection treat it as 2 seperate steps

ex $y = f(\frac{1}{2}x) \Rightarrow$ horizontal stretch by a factor of 2

$y = f(3x) \rightarrow$ horizontal compression by a factor of $\frac{1}{3}$

Adding translations into equations

$y = 2(x+3)$

① → horizontal stretch 3
② → reflect on x axis

① $y = 2(3(x+3))$

② $y = -2(3(x+3))$

$y+3 = |x-4|$

① vertical stretch of 4
② reflect on y axis

$y+3 = 4|x-4|$

② $y+3 = 4|-x-4|$

$y = x^2+3$

① reflect x axis
② reflect y axis

① $y = -x^2+3$ ② $-(-x)^2+3$

equation with multiple x's

$\dfrac{x^2-x+3}{x^3}$ reflect x $-\left(\dfrac{x^2-x+3}{x^3}\right)$

$= \dfrac{-1}{1}\left(\dfrac{x^2-x+3}{x^3}\right) = \dfrac{-x^2+x-3}{x^3}$

reflect y $\dfrac{(-x)^2-(-x)+3}{(-x)^3}$

$= \dfrac{x^2+x+3}{}$

$\boxed{3.4}$ Combining Transformations of Functions

Standard form of equation for translations

$$y-k=af(b(x-h))$$

- right
+ left

- up
+ down

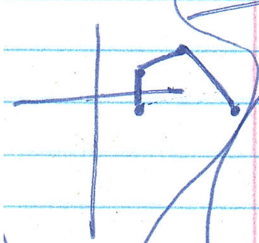
vertical
$|a|>1$ stretch
$|a|<1$ compression
$-a$ reflect on x

horizontal
$|b|>1$ compression
$|b|<1$ stretch
$-b$ reflect on y

* transformation formula
for translating points in table of values

$$\left(\frac{x}{b}+h,\ ay+k\right)$$

orig graph



ex

	f(x)
3	2
6	3
9	-2
3	-1

$\Rightarrow y+3=-2f(3(x-1))$

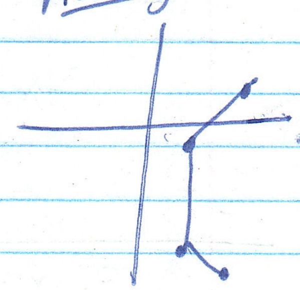
$k=-3$ $a=-2$ $b=3$ $h=1$

$\frac{x}{3}+1$	$-2y-3$
$\frac{3}{3}+1=2$	$-2(2)-3=-7$
$\frac{6}{3}+1=3$	$-2(3)-3=-9$
$\frac{9}{3}+1=4$	$-2(-2)-3=1$
$\frac{3}{3}+1=2$	$-2(-1)-3=-1$

$\boxed{\text{new table}}$ plug these values into formula.

2	-7
3	-9
4	1
2	-1

new graph



$\boxed{3.5}$ Inverse Functions

* reflection of $y=x$ line

literally x + y switch places

Table of values

$f(x)$

1	1
2	3
3	5
6	−2

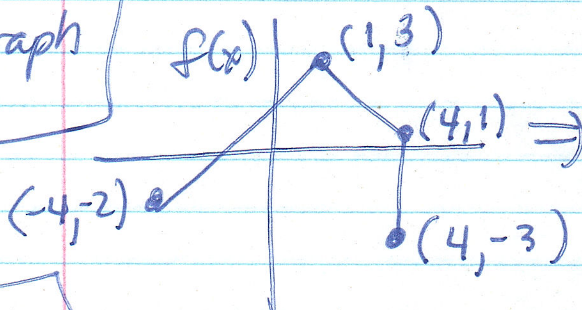
$\Rightarrow$

$f^{-1}(x)$ ← means inverse

1	1
3	2
5	3
−2	6

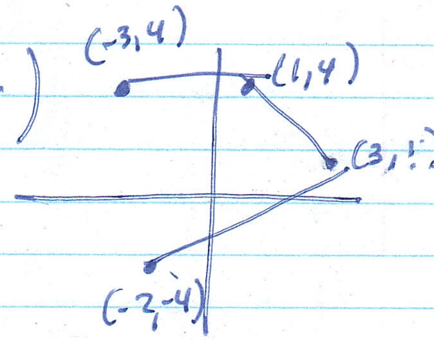
graph

$f(x)$



$\Rightarrow$

$f^{-1}(x)$



equation

ex!

$y = 3x+2 \Rightarrow$ inverse $\Rightarrow x = 3y+2$

* rearrange for y

$x-2 = 3y$

$\frac{x-2}{3} = y$

$y = -x^2+4 \Rightarrow$ inverse $\Rightarrow x = -y^2+4$

* dont forget order of operations

$x-4 = -y^2 \quad \div -1$

BEDMAS ←

$\frac{x-4}{-1} = y^2$

* remember → $\pm\sqrt{\frac{x-4}{-1}} = y$