

# CHAPTER 2: Absolute Value + Radical equations

absolute value  $\Rightarrow | \quad |$

$$|-3| = 3$$

$$|2-4|=|-2|=2$$

\* When you remove / result must be positive.

50 ...

$$\begin{aligned}
 & | -25 + 10 | - 3 | 2 - 8 | \\
 & = | -15 | - 3 | -6 | \\
 & = 15 - 3(+6) \\
 & = 15 - 18 \\
 & = -3
 \end{aligned}$$

do inside 1st

\* this means -3 times  
the result of the  
absolute value of  $|2-8|$

# radical equations

remember  $\sqrt{81} = \sqrt{9 \times 9} = 9$

$$\begin{array}{c} \sqrt{80} \\ \backslash \quad / \\ 8 \quad 10 \\ \backslash \quad / \\ 2 \quad 4 \quad 2 \quad 5 \\ \textcircled{2} \quad \textcircled{2} \quad | \\ \textcircled{2} \quad \textcircled{2} \quad | \\ \textcircled{1} \quad \textcircled{1} \quad \textcircled{1} \quad \textcircled{1} \\ \text{1 group} \quad \text{2 groups} \\ \text{of 2} \quad \text{of 2} \end{array} \quad * \text{ find groups of 2} \\ \text{then remove} \\ \text{to outside} \quad \begin{array}{l} \text{left in} \\ \text{because} \\ \text{no pair} \end{array} \quad = 4\sqrt{5}$$

rationalize the denominator:

(aka get the  $\sqrt{\phantom{x}}$  off the bottom)

Part 1

$$\frac{2\sqrt{3}}{\sqrt{2}} \quad * \text{ multiply by what the radical is on the bottom}^{\substack{\nearrow \text{both top} \\ \searrow \text{bottom}}}$$

$$\frac{2\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{2\sqrt{6}}{\sqrt{4}} = \frac{2\sqrt{6}}{2} = \text{reduce} = \sqrt{6}$$

\* remember when adding radicals  $\sqrt{\phantom{x}}$  has to be the same

So ...  $2\sqrt{3} + 4\sqrt{2} + 7\sqrt{3} - 2\sqrt{2}$

add      add      add.

$$\begin{aligned} & \underbrace{2\sqrt{3} + 7\sqrt{3}}_{\text{add}} + \underbrace{4\sqrt{2} - 2\sqrt{2}}_{\text{add}} \\ &= 9\sqrt{3} + 2\sqrt{2} \end{aligned}$$

\* Note  
 $\sqrt{x} + \sqrt{x} + \sqrt{y}$   
 $= 2\sqrt{x} + \sqrt{y}$

\* When multiplying or dividing radicals #'s with #'s, radicals w/ radicals.

mult

$$4\sqrt{2} \times 3\sqrt{5} = 4 \times 3 \text{ and } \sqrt{2} \times \sqrt{5}$$

$$= 12\sqrt{10}$$

divide

$$\frac{4\sqrt{6}}{2\sqrt{3}} = 4 \div 2 \text{ and } \sqrt{6} \div \sqrt{3} \text{ or } \frac{4}{2}\sqrt{\frac{6}{3}}$$

$$= 2\sqrt{2}$$

Rationalize the denominatorPart  
2

\* When you have + or - on bottom you must do the opposite.

$$\frac{2\sqrt{3} + \sqrt{2}}{5\sqrt{2} - \sqrt{3}} \quad \xrightarrow{\text{both top + bottom}}$$

\* mult by opposite sign on bottom

(ex  $5\sqrt{2} - \sqrt{3}$  opp sign =  $5\sqrt{2} + \sqrt{3}$ )

$$\frac{2\sqrt{3} + \sqrt{2}}{5\sqrt{2} - \sqrt{3}} \times \frac{(5\sqrt{2} + \sqrt{3})}{(5\sqrt{2} + \sqrt{3})}$$

$$= \frac{10\sqrt{6} + 2\sqrt{9} + 10\sqrt{4} + \sqrt{6}}{25\sqrt{4} + 5\sqrt{6} - 5\sqrt{6} \cancel{- \sqrt{9}}}$$

$$= \frac{10\sqrt{6} + 6 + 20 + \sqrt{6}}{25 - 3} = \frac{11\sqrt{6} + 26}{47}$$

To solve a radical equation

① get the radical alone on one side

\* remember  $\sqrt{2x+3} - 2 = 3$   
 you can't take anything out of radical before it's squared

$$\sqrt{2x+3} = 5$$

$$(\sqrt{2x+3})^2 = (5)^2$$

NOTE  
 \* once you are done check if it works (+if r → doesn't work!)

$$\begin{aligned} \sqrt{2(11)+3} - 2 &= 3 \\ \sqrt{25} - 2 &= 3 \\ 5 - 2 &= 3 \end{aligned}$$

\* once alone  $( )^2$  both sides

$2x+3 = 25$  \* now it's just algebra!!  
 $-3$   
 $2x = 22$   
 $x = 11$

# To solve an absolute value equation

\* You have to make it positive & then make it negative  
 pretend / / are brackets & put a + then - in front. (distribute)

STEP 1

$$|2x+3|=25$$

positive

$$+(2x+3)=25$$

$$2x+3=25$$

$$\begin{matrix} -3 \\ -3 \end{matrix}$$

$$2x=22$$

$$x=11$$

negative

$$-(2x+3)=25$$

$$\begin{matrix} \nearrow \\ \searrow \end{matrix}$$

$$\begin{matrix} -2x-3=25 \\ +3+3 \end{matrix}$$

$$\frac{-2x}{-2} = \frac{28}{-2}$$

$$x=-14$$

STEP 2

check if it works

$$|2(11)+3|=25$$

$$|22+3|=25$$

$$|25|=25$$

$$25=25$$

$$|2(-14)+3|=25$$

$$|-28+3|=25$$

$$|-25|=25$$

$$25=25$$

both work ✓

## Radical Equations Part 2

$$\sqrt{-3x+7} = \sqrt{-2x+9} \quad * ( )^2 \text{ both sides}$$

$$(\sqrt{-3x+7})^2 = (\sqrt{-2x+9})^2$$

$$\begin{array}{rcl} -3x+7 & = & -2x+9 \\ +2x & & +2x \end{array}$$

$$\begin{array}{rcl} -x+7 & = & 9 \\ -7 & & -7 \end{array}$$

$$-x = 2$$

$x = -2$  \* now check if it works

$$\sqrt{-3(-2)+7} = \sqrt{-2(-2)+9}$$

$$\sqrt{6+7} = \sqrt{4+9}$$

$$\sqrt{13} = \sqrt{13}$$

works ✓

example of no real roots :

NO  
REAL  
ROOTS

$$\begin{array}{rcl} \sqrt{3x-1} + 5 & = & 2 \\ -5 & & -5 \end{array}$$

$$(\sqrt{3x-1})^2 = (-3)^2$$

$$\begin{array}{rcl} 3x-1 & = & 9 \\ +1 & & +1 \end{array}$$

$$\frac{3x}{3} = \frac{10}{3} \quad x = \frac{10}{3}$$

check

$$\sqrt{3\left(\frac{10}{3}\right)-1} + 5 = 2$$

$$\sqrt{10-1} + 5 = 2$$

$$3+5 \neq 2$$

∴ NO REAL ROOTS ↴