PreCalc 11
3.1

A Factoring: a review

① look at the whole equation & decide if you can find a "common factor".

ex. $2x^2+4x-8 \Rightarrow$ 2 is a common factor.

$2(x^2+2x-4)$

② Find the factors of the front co-efficient & the end #.

ex #1

$3a^2-6a-24$ ← * means you will subtract to get the middle # & one # will be + and the other −

$3(a^2-2a-8)$ → 1 8, 2 4

$1=1\times1$

* now ask what combination would make −2

2 + 4 ⇒ Subtracted makes 2

③ Draw the 2 brackets + add signs

$3(a^2-2a-8)$

$(a \quad 2)(a \quad 4)$

2

* add or Subtract

−4

−2

* to get −2 4 has to be −

* done. $3(a+2)(a-4)$

ex #2

$3c^2 + 7c + 4$

① * nothing can factor out evenly.

② list factors

you will add

1×3 ← $3c^2 + 7c + 4$ → 1×4, 2×2

* the answer will be a combination of the factors of 3 & 4 to make +7

③ guess & check

(1 2)(3 2) → 6, 2 — NO $6+2 \neq 7$

(1 1)(3 4) → 3, 4 YES $3+4=7$

* now add in signs & variables

$(c + 1)(3c + 4)$

B If a trinomial has a decimal or fraction - get rid of it first.

$x^2 + 1.4x - 1.2$ $\div 0.1$ (same as $\times 10$)

$0.1(10x^2 + 14x - 12)$ * factor out common factors

$0.1(2(5x^2 + 7x - 6)$

$= 0.2(5x^2 + 7x - 6)$ 1×6, 2×3

1×5

↖ subtract to get 7

$(x + 2)(5x - 3)$

+10

Subract −3

+7

C If it looks like this use the

$(x+3)^2 - 6(x+3) - 16$

pretend $(x+3) = y$ & substitute

$y^2 - 6y - 16$ — 116, 2 8, 4 4

$(y+2)(y-8)$ * now plug back in

$((x+3)+2)((x+3)-8)$

$= (x+5)(x-5)$

D Remember perfect squares + difference of squares

difference of squares

$x^2 - 16$ → $(x - 4)(x + 4)$

(* has to be −)

- square root both sides + split into 2 brackets; then add a + and −

E

perfect squares

$9b^2 + 12b + 4$

- square root both side
check middle sign ⇒ if + ; then both +
if − ; then both −

$(3b + 2)(3b + 2)$

Check

$6 + 6 = 12$ ✓

F use the difference of squares if it looks like this

$$(2x-1)^2 - (y+4)^2$$

substitute: $a^2 - b^2 \Rightarrow$ factor

$$(a+b)(a-b)$$

substitute back in

$$((2x-1)+(y+4))((2x-1)-(y+4))$$

$$=(2x+y+3)(2x-y-5)$$

Do the following:

A ① $x^2+7x+10$ ② $2x^2-2x-40$

③ $5x^2+9x+4$ ④ $2x^2-9x+4$

⑤ $6x^2+x-2$

B ① $7-\frac{5}{3}x-2x^2$

③ $x^2+\frac{5}{2}x-6$

C ① $12(4x-1)^2+17(4x-1)-5$

② $3(2x-1)^2+19(2x-1)+16$

D ① $49x^2-1$ ② $4-49n^2$

E ① $25x^2+20x+4$

F ① $16(2x-7)^2-25(y+2)^2$

② $121(x+3)^2-36(2y-5)^2$

② $x^2+2.5x-1.5$

3.2

Factoring + roots (class notes)

* any equation → $x^2 + x - 2$

has a shape (+) or (−)

When you factor a trinomial you get the roots
aka x intercepts
aka zeros

for this equation

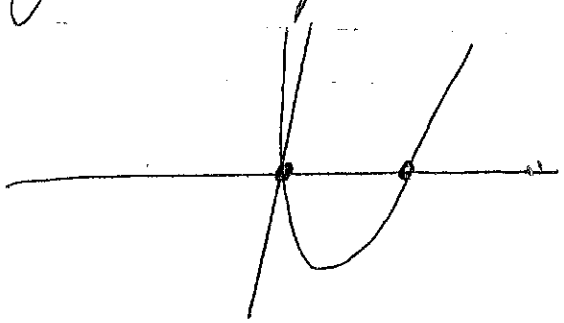
$x^2 + x - 2 \Rightarrow (x - 1)(x + 2)$

the roots are +1 and −2
in other words what would make the bracket = 0

if you had $x(x-2)$

the roots would be 0 & +2

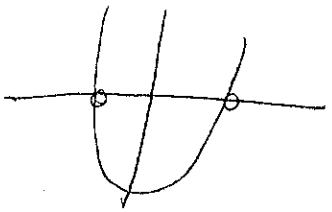
* You can plot them this way



if + ∪
if − ∩

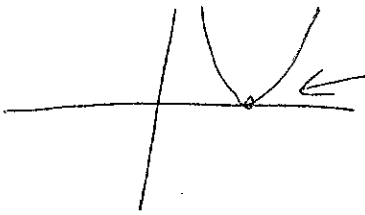
the number of roots tells you how to plot the parabola.

2 roots



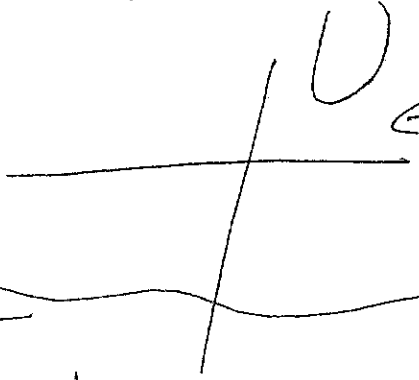
will go through x line.

2 equal roots



← doesn't pass through - bounces back up.

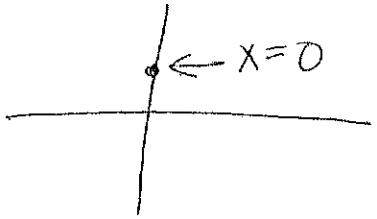
no roots



← doesn't touch x line

** get rid of $\sqrt{\ }$ $(\ )^2$ both sides

Also to add more info - you can find the y intercept by making $x=0$



x^2+x-2

$(0)^2+(0)-2$

$y\text{int} = -2$

* Do Find both x + yint

Pg. 190 # 5, 6, 7, 8, 12.

P# 197 Quiz ael

The quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow$$

View the equation you need to solve as:

$$\underline{a}x^2 + \underline{b}x + \underline{c} = 0$$

ex $3x^2 + x - 2 = 0$
[a] [b] [c]

* remember to keep the sign with the #.
* If the is no # - then it is 1.

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(1) \pm \sqrt{1^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-1 \pm \sqrt{1 + 24}}{6}$$

$$= \frac{-1 + \sqrt{25}}{6} \quad \text{and} \quad = \frac{-1 - \sqrt{25}}{6}$$

$$= \frac{4}{6} = \frac{2}{3} \qquad = \frac{-6}{6} = -1$$

∴ the roots are $\frac{2}{3}$ and -1

or $(\frac{2}{3}, 0)$ and $(-1, 0)$

(A) * if the number under the "$\sqrt{}$" doesn't factor evenly → ① change it to a decimal with your calculator
② keep in radical form.

(B) * if the number under "$\sqrt{}$" is a negative — there are no real roots.

ex A $x^2 - 6x + 7 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 28}}{2}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= \frac{6 + \sqrt{8}}{2} \qquad = \frac{6 - \sqrt{8}}{2}$$

* $\sqrt{8} = \sqrt{2 \times 2 \times 2} = 2\sqrt{2}$

reduce $\frac{6 + 2\sqrt{2}}{2}$

$= 3 + \sqrt{2}$ and $3 - \sqrt{2}$

ex B $x^2 - 5x + 7 = 0$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 - 28}}{2} = \frac{5 \pm \sqrt{-3}}{2}$$ ← cannot be negative

no real roots

Things to remember when completing the square.

1. only use the 1st 2 terms to start
2. factor out number infront of x^2
3. take ½ of the 2nd number (on x) and
   a) place inside bracket
   b) take negative outside of bracket and square it.
   c) remember to multiply negative from b by number factored from x^2 term.
4. Combine (c) with number on the outside of bracket.

ex. without factoring

$x^2+2x-46=2$

$x^2+2x-48=0$ (re write)

$\boxed{x^2+2x}-48=0$ (only 1st 2 terms)

$\left(x^2+\frac{2^2}{2}-\frac{2^2}{2}\right)-48=0$

$(x+1)^2-1^2-48=0$ (same sign as original equation)

combine

$(x+1)^2-49=0$

factoring required...

$5x^2-20x-65=-5$

$5x^2-20x-60=0$ ← re write

$5(x^2-4x)-60=0$ ← Factor 5 out of 1st 2 terms

$5\left(x^2+\frac{4^2}{2}-\frac{4^2}{2}\right)-60=0$

$\times 5$

$5(x-2)^2-(5)(2^2)-60=0$

$5(x-2)^2-20-60=0$

$5(x-2)^2-80=0$

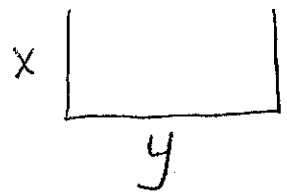
Max/Min Problems:

Method 1:

* hint: draw it out & label sides x & y

① write the equations from the word problem
② rearrange one of the equations to find max/min ← plug into max/min equats.
③ expand
④ complete the square
⑤ $a(x-p)+q \Rightarrow -p$ value is answer for x, q is the max/min

ex. A rectangular lot has one side along a river. The other 3 sides have a total of 80m of fencing. What is the max area?



① equation #1 $2x+y=80m$

max equation → $xy=\text{max}$

② $2x+y=80 \Rightarrow 2x-80=-y \Rightarrow -2x+80=y$

$xy=\text{max}$; $x(-2x+80)=\text{max}$

③ $-2x^2+80x=\text{max}$

④ $-2(x^2-40x)$; $-2(x^2-(20)^2+20^2)$;
$-2(x-20)^2-20^2x-2$

$=-2(x-20)^2+800$

(x value; ↖ max area)

∴ the max area is $800\,m^2$
the x value (width) is 20
so the y value (length) $= 2(20)+y=80$; $y=40$.

Max/Min Problems:

Method 2 → use when trying to find revenue when # increases or decreases because of price.

$R = (\text{cost})(\text{number})$

ex. Computer company sells software to students for $20. Three hundred students buy the software. For every $5 increase in price, 30 less students buy the software.

Find max revenue & max price.

① $R = (20 + x)(300 - 30(\frac{x}{5}))$ (orig. cost; orig. # sold; # less; per $5 increase)

② expand ⇒ $(20+x)(300-6x)$
$= -6x^2 + 180x + 6000$

③ complete the square ⇒ $-6(x^2 - 30x) + 6000$
$-6(x^2 - 15^2 + 15^2) + 6000$
$-6(x-15)^2 + 6000 - 15^2 x - 6$
$= -6(x-15)^2 + 7350$

↑ increase to get max revenue

↑ max revenue

∴ price for max revenue is $\$20 + \$15 = \$35$.