CHAPTER 1 - PCMATH 12 notes

1.1 Dividing a Polynomial by a Binomial

To divide → ① write exponents in descending order

ex $-x+3x^3-6+2x^2 \Rightarrow 3x^3+2x^2-x-6$

② write as a long division question

ex $3x^3+2x^2-x-6 \div x+2 \Rightarrow x+2\,\overline{)\,3x^3+2x^2-x-6}$

③ calculated what you need to multiply 1st term (divisor) to get the 1st term in the polynomial.

$$\begin{array}{r} 3x^2 \\ \underline{x}+2\,\overline{)\,\underline{3x}^3+2x^2-x-6} \end{array}$$

$x \cdot \boxed{3x^2} = 3x^3$

④ multiply both terms in divisor by this monomial and subtract.

$$\begin{array}{r} 3x^2-8x+7 \\ x+2\,\overline{)\,3x^3+2x^2-x-6} \\ \ominus\ 3x^3+6x^2\quad\downarrow \\ \hline -4x^2-x \\ \ominus\ -4x^2-8x\quad\downarrow \\ \hline 7x-6 \\ \ominus\ 7x+14 \\ \hline -20 \end{array}$$

$x \cdot 3x^2 = 3x^3$

$2 \cdot 3x^2 = 6x^2$

* careful $\bar{w}$ + + −

* repeat above steps

$$\boxed{\begin{array}{l} x \cdot -4x = -4x^2 \\ 2 \cdot -4x = -8x \end{array}}$$

$x \cdot 7 = 7x$

$2 \cdot 7 = 14$

$\boxed{\therefore \text{quotient} = 3x^2-8x+7 \text{ R } -20}$

* if you have a space between exponents fill in with a 0.

ex $-4x^4+2x^2-x-3 \Rightarrow -4x^4+0x^3+2x^2-x-3$

Synthetic division: $x-a$; bx^2+cx+d

$x-2$; $\underline{5}x^2\,\underline{+7}x\,\underline{-4}$

ex1

$$\begin{array}{c|ccc} 2 & 5 & 7 & -4 \\ & \downarrow & \boxed{10} & \boxed{34} \\ \hline & \underbrace{5 \quad 17}_{\text{quotient}} & & \underbrace{30}_{R} \end{array}$$

→ 17 x 2 = 34 then add

→ 5 x 2 = 10 then add

∴ quotient is $5x+17$ R30

ex2 $2x^3+4x^2-5x-6$; $x\underline{+1}$

$$\begin{array}{c|cccc} -1 & 2 & 4 & -5 & -6 \\ & & (2\cdot-1) & (2\cdot-1) & (-7\cdot-1) \\ & \downarrow & -2 & -2 & 7 \\ \hline & 2 & 2 & -7 & 1 \end{array}$$

∴ quotient is $2x^2+2x-7$ R1

$\boxed{1.2}$ Factoring Polynomials

use $x - a \leftarrow$ if $(x+a) \rightarrow$ think of it as $x-(-a)$

Remainder theorem – when a polynomial $(P(x))$ is divided by $x-a$ the remainder is $P(a)$

ex $\boxed{\text{using long division}}$

$$
\begin{array}{r}
5x^2+8x+24 \\
x-2\,\overline{)\,5x^3-2x^2+8x-1} \\
\ominus\; 5x^3-10x^2 \quad\downarrow \\
\hline
8x^2+8x \\
\ominus\; 8x^2-16x \\
\hline
24x-1 \\
24x-48 \\
\hline
R=47
\end{array}
$$

using the $\boxed{\text{Remainder Theorem}}$

$5x^3-2x^2+8x-1 \div x-2$

$\overset{\uparrow}{a=2}$

$5(2)^3-2(2)^2+8(2)-1$

$=40-8+16-1$

$=47$

$R=47$

Factor theorem – if the remainder $=0$ it is a factor

---

ex $3x^4+7x^3-x^2+14x-3$

$\div x-1$

$3(+1)^4+7(1)^3-(1)^2+14(1)-3$

$=20$

NOT A FACTOR

$\div x+3$

$3(-3)^4+7(-3)^3-(-3)^2+14(-3)-3$

$=0$

IS A FACTOR

Why do you want to know if it a factor?

① to find roots / zeros

② to simplify complex equations

To factor $2x^3-9x^2+7x+6$

Guess & check - if R=0, it is a factor

↑ the factors P(x) will have the same factors as the last # (factor property)

try $x-1$;

$$2(1)^3-9(1)^2+7(1)+6$$

$$= 2-9+7+6$$

$$= 6$$ NOT A FACTOR

try $x-2$

$$2(2)^3-9(2)^2+7(2)+6$$

$$= 16-36+14+6$$

$$= 0$$ * IS A FACTOR

* once you find a factor - divide P(x) by it

$$\begin{array}{r|l}
 & \phantom{2x^3-{}}2x^2-5x-3 \\
\hline
x-2 & 2x^3-9x^2+7x+6 \\
\ominus & 2x^3-4x^2\downarrow \\
\hline
 & -5x^2+7x \\
\ominus & -5x^2+10x \\
\hline
 & -3x+6 \\
 & -3x+6 \\
\hline
\end{array}$$

or

$$\begin{array}{r|rrrr}
2 & 2 & -9 & 7 & 6 \\
 & & 4 & -10 & -6 \\
\hline
 & 2 & -5 & -3 & 0
\end{array}$$

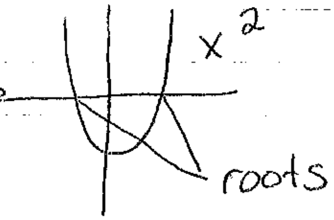
so... $2x^3-9x^2+7x+6^{0} = (x-2)(2x^2-5x-3)$

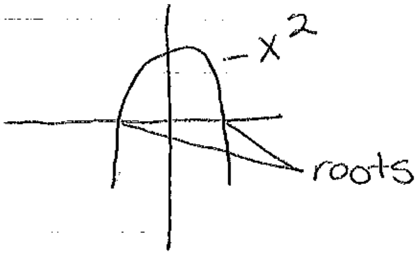
$= (x-2)(2x+1)(x-3)$ ↑factor

$\boxed{11.3}$ Graphing Polynomial Functions

X^2

=2 directions

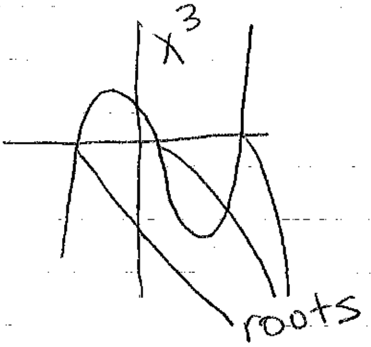


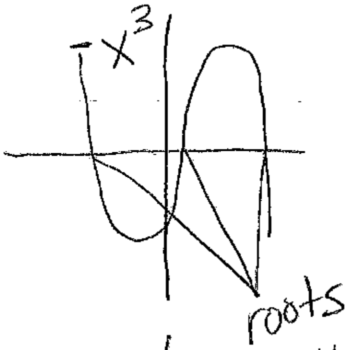


$$x^2+2x\underbrace{-3}_{\text{y intercept}}$$

X^3

=3directions

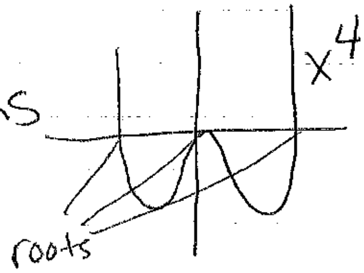


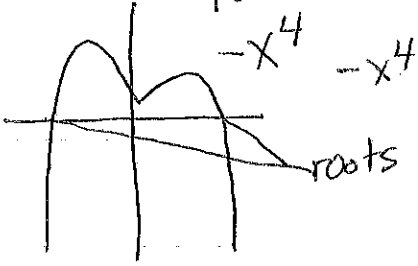


$$2x^3+3x^2-3x\underbrace{-2}_{\text{y intercept}}$$

X^4

-4directions

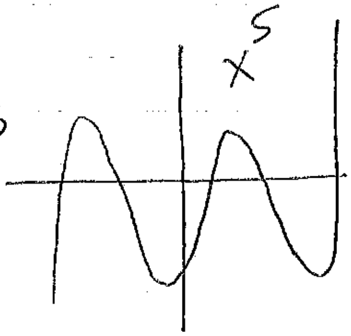


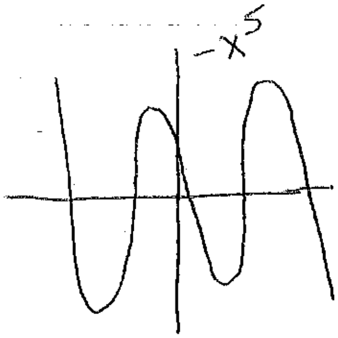


$$-x^4+3x^3+4x^2-12x\underbrace{-2}_{\text{y intercept}}$$

X^5

-5directions





$$x^5+2x^4-7x^3-8x^2+12x\underbrace{-1}_{\text{y intercept}}$$

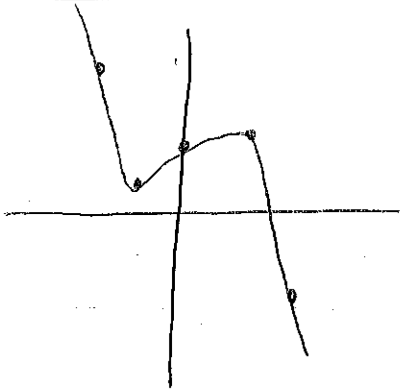
$\boxed{1.4}$ Relating Polynomial Functions and Equations

degree = leading / highest power

Graphing - $\boxed{\text{Method 1}}$; table of values

$-2x^3+4x+4$

x	y
-2	12
-1	2
0	4
1	6
2	-4



Graphing - $\boxed{\text{Method 2}}$ - factoring

$2x^4-x^3-14x^2+19x-6$

① try $x-1$:> $2(1)^4-x(1)^3-14(1)^2+19(1)-6=0$

So $x-1$ if a factor - now divide

②

$$\begin{array}{r} 2x^3+x^2-13x+6 \\ x-1\,\overline{)\,2x^4-x^3-14x^2+19x-6} \\ \ominus\ \underline{2x^4-2x^3} \downarrow \\ x^3-14x^2 \downarrow \\ \ominus\ \underline{x^3-x^2} \\ -13x^2+19x \\ \ominus\ \underline{-13x^2+13x} \\ 6x-6 \\ \underline{6x-6} \\ 0 \end{array}$$

$(x-1)(2x^3+x^2-13x+6)$

③ find factor for this

try $x-2$

$2(2)^3+(2)^2-13(2)+6=0$

④

$$\begin{array}{r} 2x^2+5x-3 \\ x-2\,\overline{)\,2x^3+x^2-13x+6} \\ \ominus\ \underline{2x^3-4x^2} \downarrow \quad \downarrow \\ 5x^2-13x \\ \ominus\ \underline{5x^2-10x} \\ -3x+6 \\ \underline{-3x+6} \\ 0 \end{array}$$

next page ....

1.4

now $2x^4-x^3-14x^2+19x-6$

$= (x-1)(x-2)(2x^2+5x-3)$

↖ factor ⑤

$(2x-1)(x+3)$

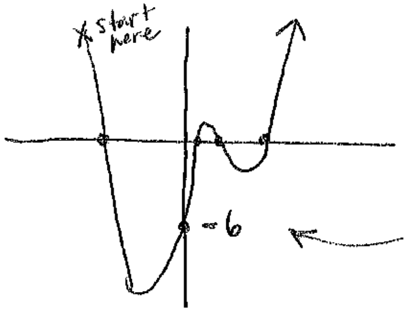
∴ roots / factors are

$(x-1)(x-2)(2x-1)(x+3)$

⑥ and roots $+1$; $+2$; $+\frac{1}{2}$; -3

__graph__ $2x^4-x^3-14x^2+19x-6$ ← y intercept

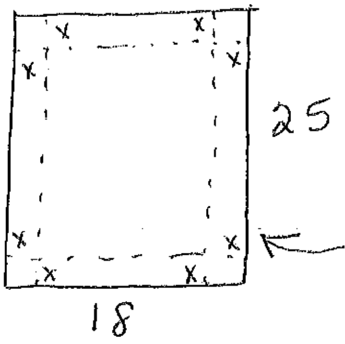
positive so W



$\boxed{1.5}$ Modelling & Solving Problems w Polynomial Functions

* DRAW IT OUT !

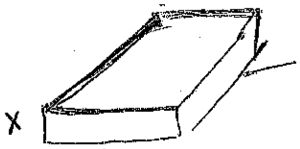
max volume : 25 long 18 wide



- a pieee is cut at corners & folded up

you dont know length of this piece so it is X.

now draw out the folded box



used to be 25 but now x is taken off both sides→(25-2x)

used to be 18 but now x + x is foldedup so now (18-2x)

$V = l \times w \times h$

$= (25-2x)(18-2x)(x)$

Graphing Calculator :

$y=(25-2x)(18-2x)(x)$

2nd CALC → MAX

X = X value

y = max. volume

* $x \geq 0$ can't have a side as -1 !

1.5

ex 2

Leo and 3 friends

Leo $= x$; Sandra 3 years younger $= x-3$
Vince 4 years older $= x+4$
Hunter 1 year older $= x+1$

Find ages if product of ages $= 54658$ more than sum of ages

product of ages $= (x)(x-3)(x+4)(x+1)$

Sum of ages $= (x)+(x-3)+(x+4)+(x+1)$
$= 4x+2$

$$(x)(x-3)(x+4)(x+1) = 4x+2+54658$$

* move everything to one side to $= 0$ then graph; where $y=0$ is your answer

$$0=(x)(x-3)(x+4)(x+1)-4x-54660$$

* remember Leo's age cannot be ≤ 0

- on the graph $y=0$ at 15

so ∴ Leo $= 15$
Sandra $= 15-3 = 12$
Vince $= 15+4 = 19$
Hunter $= 15+1 = 16$