CH4 - PCMA12

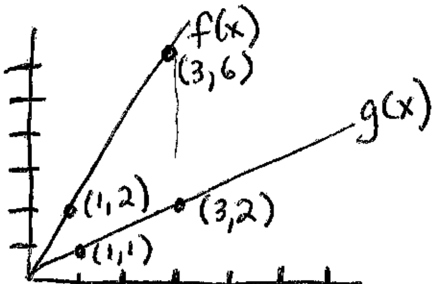
$\boxed{4.1}$ Combining Functions Graphically

$f(x) + g(x)$; $f(x) - g(x)$

$f(x) \div g(x)$; $f(x) \cdot g(x)$

* remember $f(x) \Rightarrow$ is the same thing as saying the y value or answer

$g(x)$ or $h(x)$ .... are just other functions different than $f(x)$

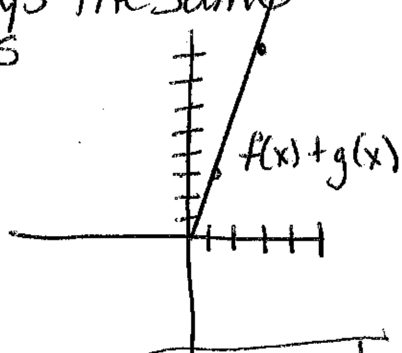


$\boxed{f(x) + g(x)}$

x value stays the same
add y values

$f(x) + g(x)$

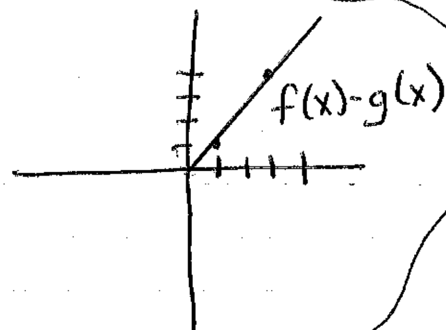
x	y
1	1+2=3
3	2+6=8



$\boxed{f(x) - g(x)}$

x value stays the same
subtract y values

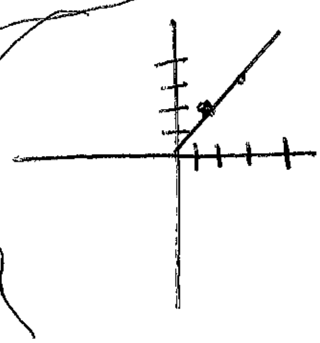
x	y
	f(x) - g(x)
1	2-1=1
3	6-2=4



$\boxed{f(x) \div g(x)}$

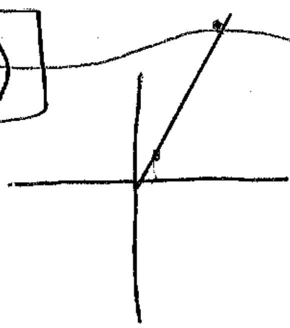
x = same
÷ f(x) by g(x)

x	y
1	2÷1=2
3	6÷2=3



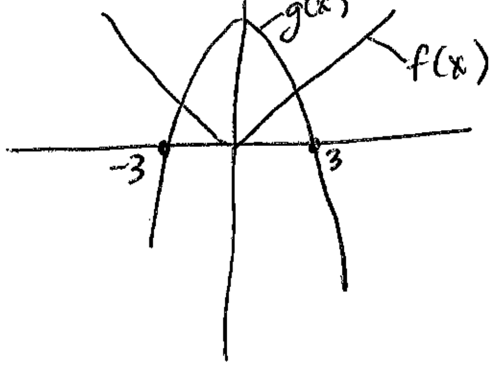
$\boxed{f(x) \cdot g(x)}$

x	y
1	2×1=2
3	6×2=12



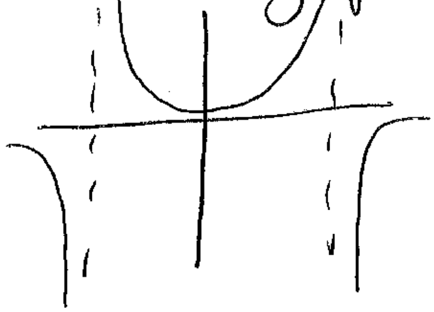
* Remember when dividing the denominator cannot be zero.

To combine

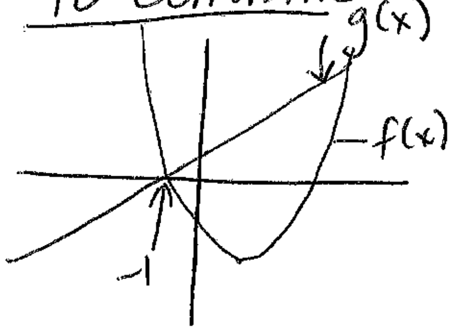


$$\frac{f(x)}{g(x)} \Rightarrow \text{at } 3 \text{ or } -3 = \frac{f(x)}{0}$$

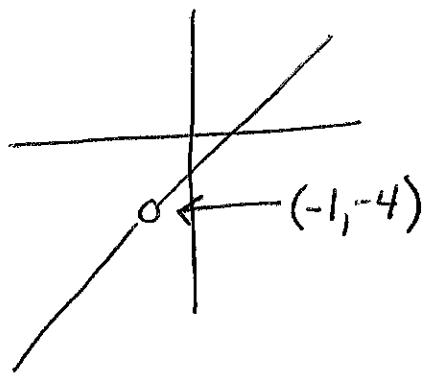
* draw asymptotes



To combine



$$\frac{f(x)}{g(x)} \Rightarrow x \neq -1 \text{ or } \frac{f(x)}{0}$$



* if the Inverse in a line draw an open circle or 'hole'

*

4.2 Combining Functions Algebriacally

To combine functions + find the new function – follow the order of operations

ex. $f(x) = x^2$ $g(x) = 2x - 3$

$f(x) + g(x) \Rightarrow x^2 + 2x - 3$

$f(x) - g(x) \Rightarrow x^2 - (2x-3)$
$= x^2 - 2x + 3$

* remember to Subtract the entire function
use brackets

$f(x) \div g(x) \Rightarrow \dfrac{x^2}{2x-3}$

$f(x) \cdot g(x) \Rightarrow x^2(2x-3)$
$= 2x^3 - 3x^2$

$\boxed{4.3}$ Composite Functions * START INSIDE OUT!

* functions are imbedded into each other.

ex f(x) happens 1st; then using this answer to find g(x)

$X \to \overset{(A)}{\boxed{2x+3}} \to \overset{(B)}{\boxed{\frac{1}{x}}}$

if $x = 2$

step1 $\to 2(2)+3 = 7$

Step 2 (x is now $= 7$)

$\frac{1}{7}$

so $g(f(x)) = \frac{1}{7}$

ex2 what if the machine went backwards and did $f(g(x))$ or $f \circ g(x)$

$x=2 \quad g(x) = \frac{1}{2}$

$f(x) = \cancel{2}(2(\frac{1}{2})+3)$

$= 1+3 = 4$

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you can use a table of values

f(x)

x	y
0	3
1	5
2	7

g(x)

2	$\frac{1}{2}$
5	$\frac{1}{5}$
7	$\frac{1}{7}$

$g(f(x))$

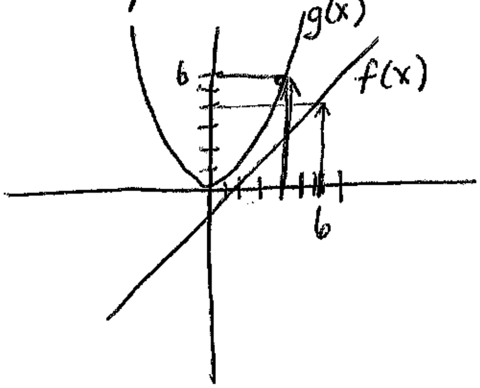
start

$x=1 \Rightarrow f(x)=5$

now plug into g(x)

$x=5 \therefore g(f(x)) = \frac{1}{5}$

Composite – using a graph



find $f(g(4))$

① $g(4) = 6$

② now ask what is $f(x)$ at $x = 6$

$f(6) = 4$

③ so $\therefore$ $f(g(4)) = 4$

* remember $f(x) = y$ value

$f(4)$

↑

x value

ex $f(x) = |4 - x|$; $g(x) = (x-4)^2$; $h(x) = \sqrt{x}$

find $h(g(f(2)))$

start here + work out

① $f(2) = |4-2| = 2$ (now use this for $g(x)$)

② $g(2) = (2-4)^2 = 4$ (now use this for $h(x)$)

③ $h(4) = \sqrt{4} = 2$

$\therefore h(g(f(2))) = 2$

$\boxed{4.4}$ Restrictions on Composite Functions

ex Restriction

$\frac{1}{x+3} \rightarrow x \neq -3$ $\quad\quad$ $\sqrt{x} \Rightarrow x \geq 0$

If a function has a restriction and is placed in the centre

$f(g(x))$

The restriction applies to the composite

ex 1

$f(x) = \sqrt{x}$ $\quad x \geq 0$

$g(x) = x+3$ $\quad$ no restrictions

$g(f(x))$ automatically has the restriction $x \geq 0$

To find other restrictions use algebra

$f(g(x)) = \sqrt{x+3}$ $\quad$ since $\cancel{\sqrt{x}}$

* new restriction $x \geq -3$

but x can be ≤ 0 but not < -3

ex 2 $f(x) = \frac{1}{x-2}$ $\quad x \neq 2$

$g(x) = x^2 - x$ no restriction

$g(f(x)) = \left(\frac{1}{x-2}\right)^2 - \frac{1}{x-2}$

original restriction applies $x \neq 2$

$f(g(x)) = \frac{1}{x^2-x-2} = \frac{1}{(x+1)(x-2)}$ $\quad x \neq -1; 2$