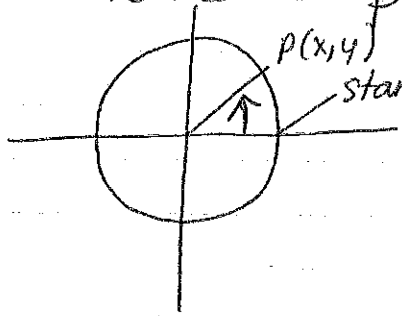
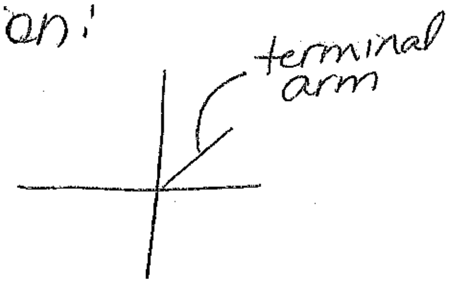
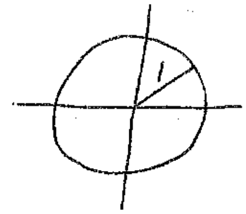
Chapter 6 notes:

6.1 Standard position:

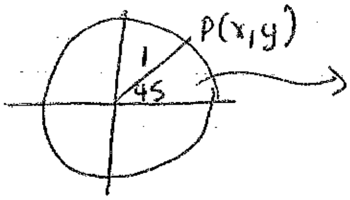


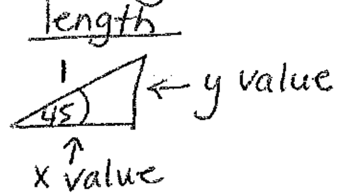


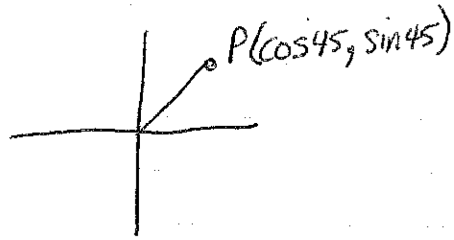
* unit circle because r=1

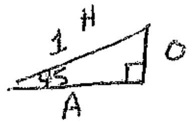


To find the length of a terminal arm or a point: use trigonometry







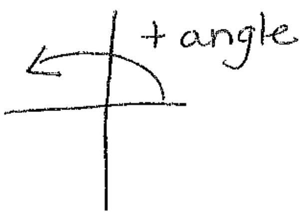


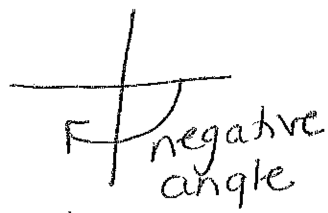
$\sin 45 = \frac{O}{H}$ $\qquad \sin 45 = \frac{y}{1} \sim y = \sin 45$

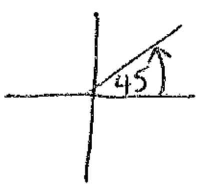
$\therefore$ y value =

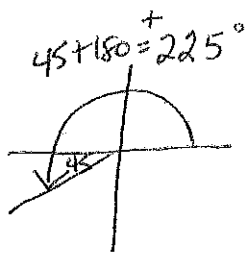
$\cos 45 = \frac{A}{H}$ $\qquad \cos 45 = \frac{x}{1}$ $\therefore$ x value =

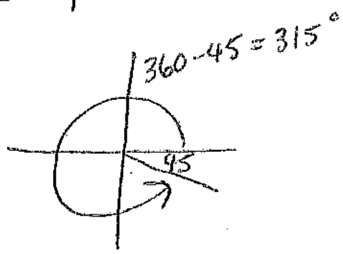
P(,)

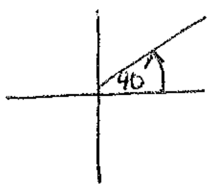


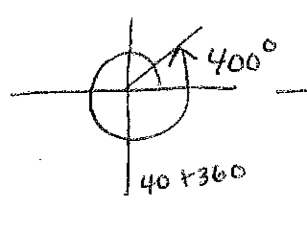


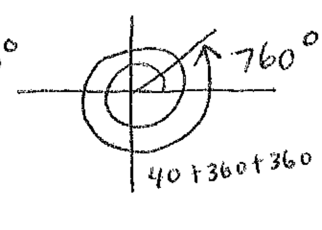












reference angles : copy pg 476

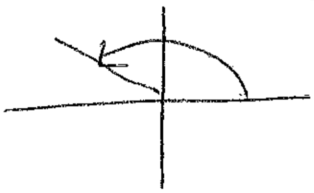
$\measuredangle\ 510^\circ \rightarrow 510 - 360 = 150$
(one circle)

Q2	Q1
SIN + cos − tan −	SIN + cos + tan +
sin − cos − tan + Q3	sin − cos + tan − Q4

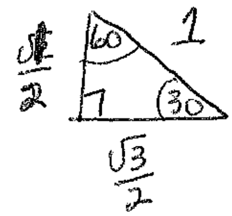
← * remember (cos, sin)
(x , y)

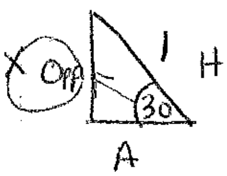
in quad 1 cos + ; x +
in quad 2 cos − ; x −
in quad 3 cos − ; x −
in quad 4 cos + ; x +

So 150° is 30°



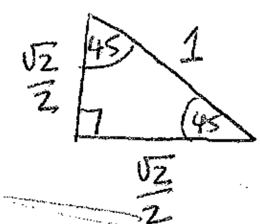
or $-\cos 30^\circ$
and $+\sin 30^\circ$





$\sin 30 = \frac{X}{1}$ or $\frac{1}{2}$

$-\cos 30 = \frac{A}{1} = -\frac{\sqrt{3}}{2}$



$x^2 + y^2 = r^2$ given $\csc\theta = 3$

$\sin = \frac{y}{r}$ $\therefore$ $\csc = \frac{r}{y} = \frac{3}{1}$

$x^2 + y^2 = r^2$

$x^2 + (1)^2 = 3^2$
$\quad -1 \qquad -1$

$x^2 = 9 - 1$ $x = \pm\sqrt{8}$

$y = 1$; $r = 3$, $x = \pm\sqrt{8}$

now $\cos\theta = \frac{x}{r} = \frac{\pm\sqrt{8}}{3}$

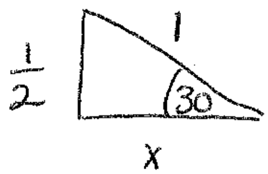
$\tan = \frac{\pm\sqrt{8}}{1}$ $\sec = \frac{r}{x} = \frac{3}{\pm\sqrt{8}}$

$\cot = \frac{1}{\pm\sqrt{8}}$

To draw triangles:

Start with what you know:

unit circle $H=1$ because $r=1$

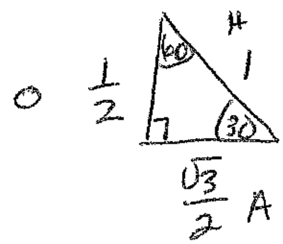


$\sin 30 = 0.5$ or $\frac{1}{2}$

To FIND x use pyth. theorem

$$\sqrt{1^2-\left(\frac{1}{2}\right)^2}=\sqrt{\frac{4}{4}-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

So now the triangle looks like this:



$\boxed{30^\circ}$

$\sin 30 = \frac{\frac{1}{2}}{1}$ or $\frac{1}{2}$

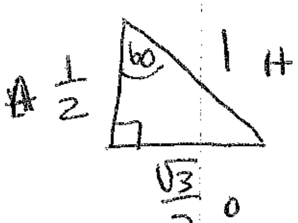
~~cos30=~~

$\cos 30 = \frac{\frac{\sqrt{3}}{2}}{1}$ or $\frac{\sqrt{3}}{2}$

$\tan 30 = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

or $\frac{1\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\boxed{60^\circ}$



$\sin 60 = \frac{\frac{\sqrt{3}}{2}}{1}$ or $\frac{\sqrt{3}}{2}$

$\cos 60 = \frac{\frac{1}{2}}{1}$ or $\frac{1}{2}$

$\tan 60 = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \div \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

$\boxed{45^\circ}$

~ if both angles are the same then both sides are the same ...

$a^2+b^2=c^2$ (same)

$x^2+x^2=(1)^2$

$2x^2=1^2$

$x^2=\frac{1}{2} \Rightarrow \sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$\boxed{90^\circ}$

$\sin 90^\circ = +1$
$\cos 90^\circ = 0$
$\tan 90 = \text{undefined}$

same just different quadrants

$\sin 270 = -1$
$\cos 270 = 0$
$\tan 270 = \text{undefined}$

$\sin 180^\circ = 0$
$\cos 180^\circ = -1$
$\tan 180^\circ = 0$

same just diff quadrant

$\sin 360^\circ = 0$
$\cos 360^\circ = 1$
$\tan 360^\circ = 0$

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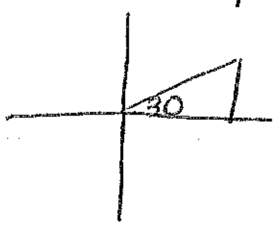
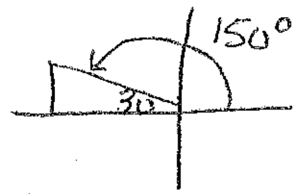
other angles ....

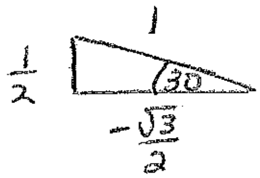


$\Rightarrow$



$180 - 30 = 150$

Same values just different quadrant



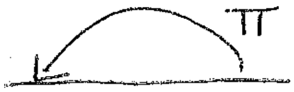
So $\sin 150 = \frac{1}{2}$

$\cos 150 = -\frac{\sqrt{3}}{2}$

$\tan 150 = -\frac{1}{\sqrt{3}}$

$\boxed{6.2 \text{ \& } 6.3}$

Radian and arc length:



– count # of segments (#) then ÷ π by #

$\frac{180}{6} = \frac{\pi}{6}$

To find radian length:

1 radian $= \frac{180°}{\pi}$ $\qquad$ π radians $= 180°$

$1° = \frac{\pi}{180}$ radians $\qquad$ $360° = 2\pi$ radians

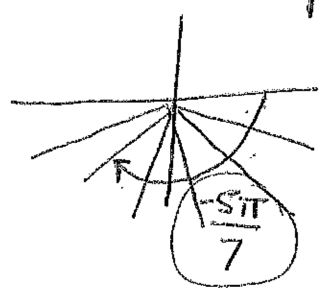
(I) given degrees → find radians

$\theta = 255° \Rightarrow 255 \times \frac{\pi}{180} = \frac{17}{12}\pi$ radians or 4.45 radians

(II) given radians → find degrees

$-\frac{5\pi}{7} \Rightarrow -\frac{5\pi}{7} \times \frac{180}{\pi} \doteq -128.6°$

to sketch → keep in radians

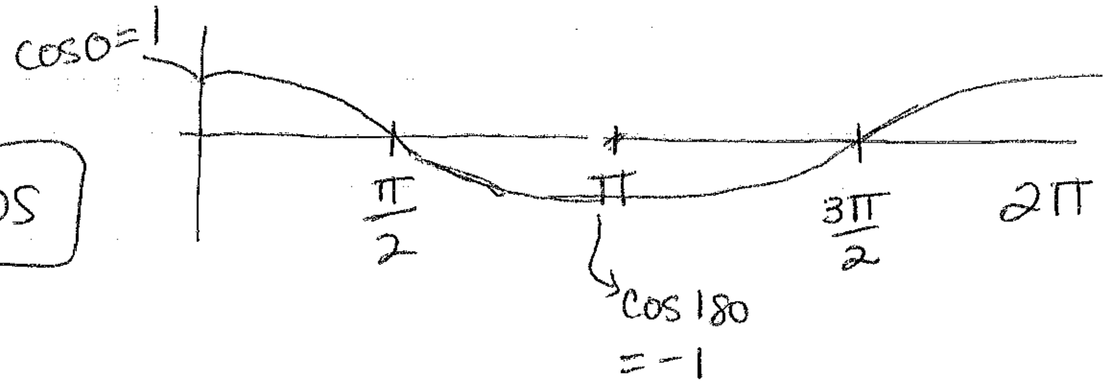


↙ ÷ into 7 parts & go backward 5

$\boxed{\text{COS}}$

		30°	45°	60°	90°				
X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
COS X	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	-1

$\boxed{\text{Cos}}$

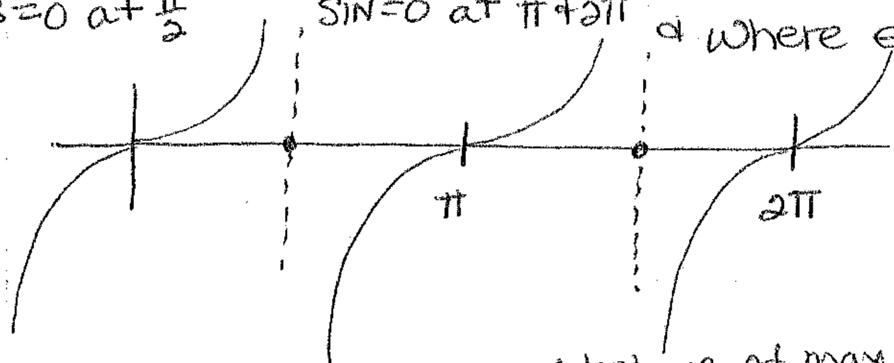


$\boxed{\text{TAN}}$

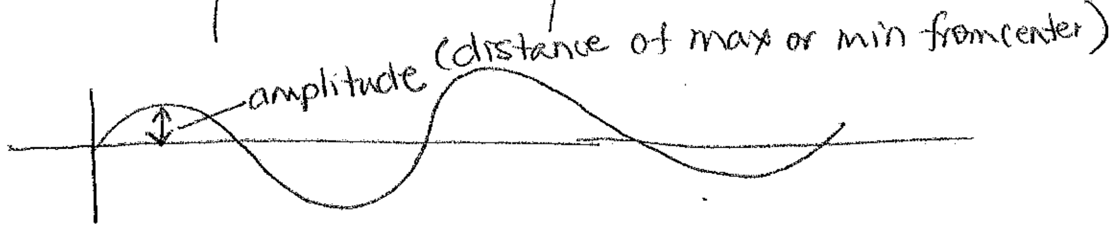
Since $\text{Tan} = \frac{\text{SIN}}{\text{COS}}$; where ever cos = 0 there is an asymptote & where ever sin = 0 it crosses.

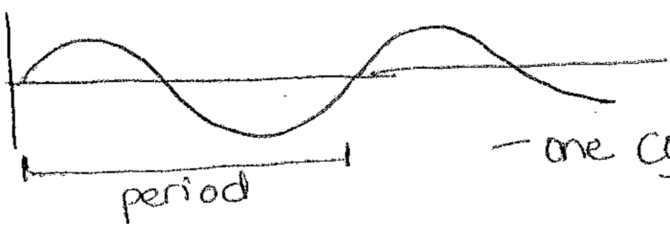
Cos=0 at $\frac{\pi}{2}$

SIN=0 at $\pi + 2\pi$



$\boxed{6.5}$





– one <u>complete</u> cycle

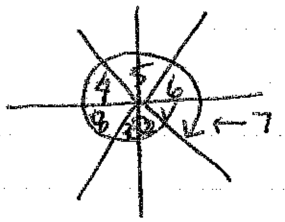
To determine $\underline{\text{exact}}$ values: $\tan\left(-\frac{7\pi}{3}\right)$

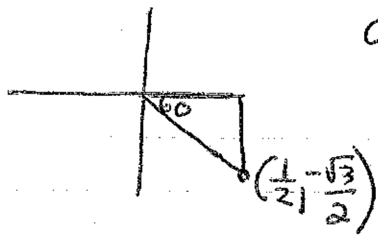
① remind yourself of exact values for sin, cos from 6.1,

② Draw on the unit circle

$-\frac{7\pi}{3}$

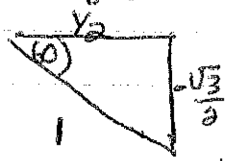
÷ 180 into 3 parts





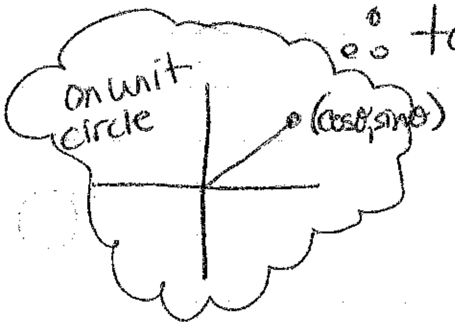
ask – what would be the measure of this angle?

$180 \div 3 = 60°$



now label

neg. because in quad 4 ⇒ y value is negative here.



$\therefore \tan 60 = \frac{O}{A}$

$$= \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = \frac{-\sqrt{3}}{2} \times \frac{2}{1} = -\sqrt{3}$$

③ To determine in decimal form (nearest hundredth)

*change calculator settings to RADIANS → then put in

$$\tan\left(\frac{-7\pi}{3}\right) = -1.73$$

④ To find arc length $a = r\theta$

* calculator should be in RADIANS *

Ⓐ given arc length = 90 & find angle and r = 36

$90 = 36(\theta)$ so $\theta = \frac{90}{36} \Rightarrow 2.5$ radians or $2.5 \times \frac{180}{\pi} \doteq 143°$

(B) given radius and angle ⇒ find arc length

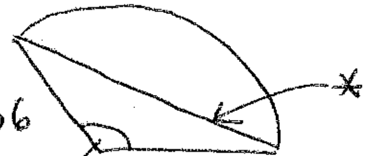
$r = 5 \quad \theta = 60^\circ \Rightarrow a = r\theta$

* ↳ convert to radians

$60 \times \frac{\pi}{180} = 1.05$ radians

$a = (5)(1.05)$
$= 5.23$

(C) FIND A DISTANCE : * radian mode *



* calculated in (A) to be 2.5 radians

⇒ use cos law

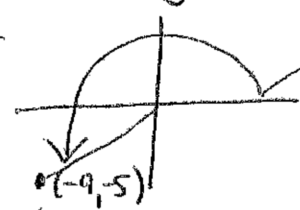
$b^2 = a^2 + c^2 - 2ac\cos B$

$b = \sqrt{36^2 + 36^2 - 2(36)(36)\cos 2.5}$
$= 68.32$

(5) Find angle using <u>terminal point</u>

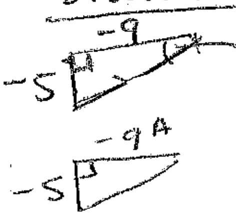
given (-9,-5)

<u>Draw it out</u>

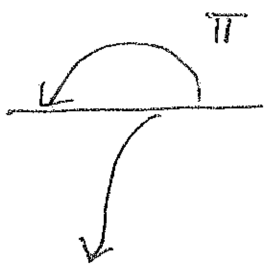


always start in standard position

<u>Draw △</u>



this angle



*radian mode

$\tan\theta = \frac{-5}{-9}$

$= 0.5070 + \pi \doteq 3.65$ or $3.65 \times \frac{180}{\pi} = 209^\circ$

6.4 Graphing Trig Functions:

SIN

* radian mode

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

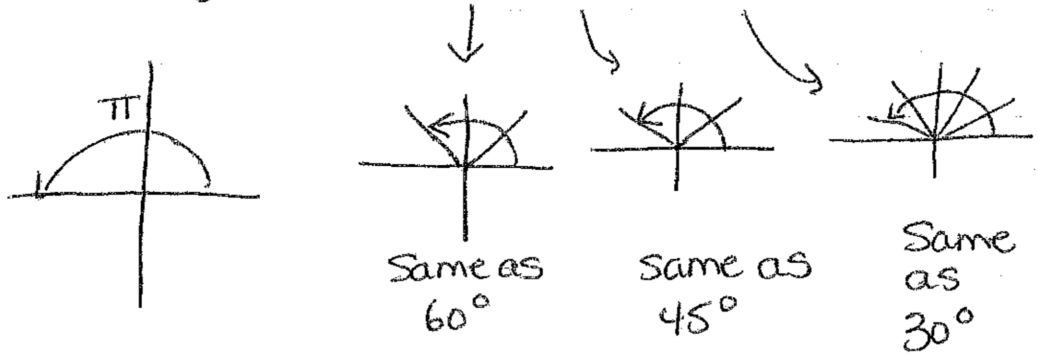
remember

$\frac{\pi}{2} = 90°$

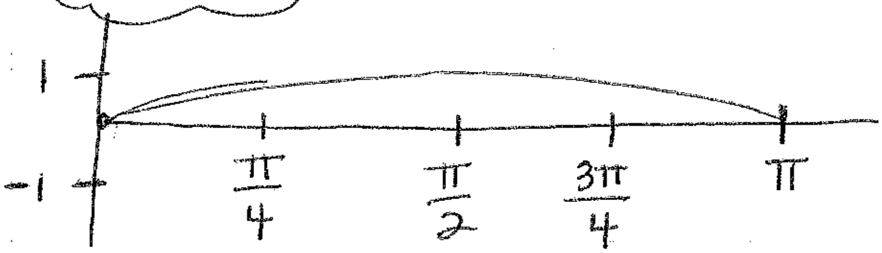
$\frac{\pi}{3} = 60°$

$\frac{\pi}{4} = 45°$

$\frac{\pi}{6} = 30°$

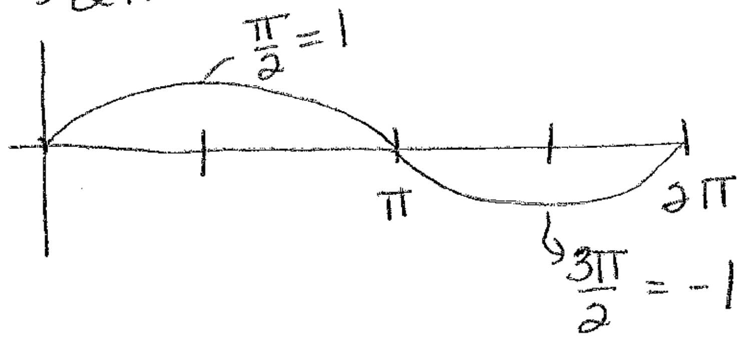


$0 \to \pi$



$0 \to 2\pi$

SIN



$\boxed{\text{transformations in amplitude}}$ $y = A\cos x$

$y = 3\cos x$ → orig. amplitude is 1

so new amplitude $= 3 \times 1 = 3$

$y = -\frac{1}{2}\sin x$ → orig. amplitude is 1

so new amplitude $= \frac{1}{2} \times 1 = \frac{1}{2}$

reflected on x axis.

$\boxed{\text{transformations in period}}$ $y = \cos Bx$

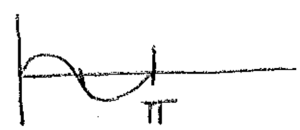
<u>period changes</u> ⇒ $y = \sin bx$, $y = \cos bx$ ⇒ $\frac{2\pi}{b}$

⇒ $y = \tan bx$ ⇒ $\frac{\pi}{b}$

$y = \sin 2x$

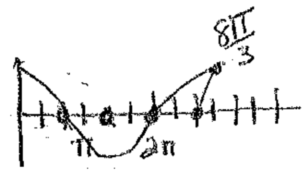
→ original period $= 2\pi$

new period $= \frac{2\pi}{2} = \pi$



$y = \cos \frac{3}{4}\pi$ → original period $= 2\pi$

new period $= \frac{2\pi}{\frac{3}{4}}$ or $2\pi \times \frac{4}{3} = \frac{8\pi}{3}$



$\frac{1}{2}$ way is $\frac{4\pi}{3} = -1$

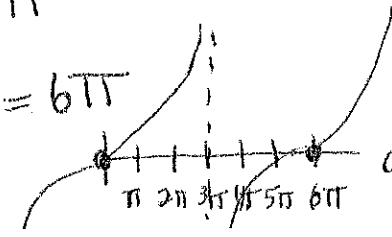
$\frac{1}{4}$ way $= \frac{2\pi}{3}$

and $\frac{3}{4}$ way $= \frac{6\pi}{3}$

$y = \tan\frac{x}{6}$ ⇒ $\tan\frac{1}{6}x$

original period $= \pi$

new period $= \frac{\pi}{\frac{1}{6}}$ or $\pi \times \frac{6}{1} = 6\pi$



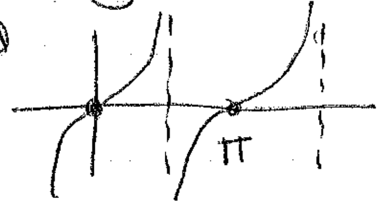
asymptote $= \frac{1}{2}$ way

<u>phase shift</u> $y = \tan(x + b)$ ← to left

or $y = \tan(x - b)$ ← to right

$y = \tan(x + \frac{\pi}{2})$ → move graph $\frac{\pi}{2}$ to left.

original tan

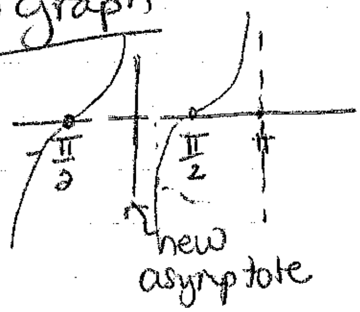


original points

$\tan 0 = 0$ $\tan \frac{\pi}{2} = \text{undef}$

$\tan \pi = 0$ $\tan \frac{3\pi}{2} = \text{undef.}$

<u>new graph</u>



$\boxed{6.6}$ <u>Combining</u> transformations:

$y = af(b(x - c)) + d$ → − down, + up

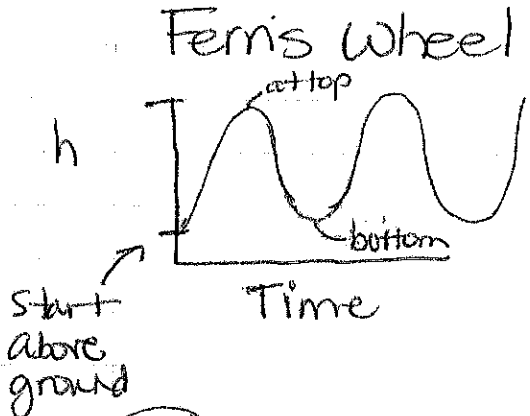
Change in amplitude × a

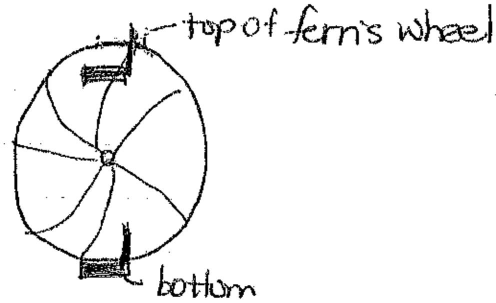
change in period $\frac{2\pi}{b}$ or $\frac{\pi}{b}$

c: + left − right

* do 1st → then translations

6.7 Applications of Sinusoidal Functions.





ex.1 ferris wheel $d = 150m$ 1 revolution $= 32$ min
person reaches max height of 165m

this means → period = 32 min
→ the graph starts 15m above ground
→ max height is 165 at $\frac{32}{2}$ min

So far the points we have are:
$(0,15)$, $(16,165)$, $(32,0)$

To find the $\boxed{\text{centre line}}$ $\frac{\overset{\text{max + min}}{165+15}}{2} = 90$

To find $\boxed{\text{amplitude}}$:
take max height − centre
$165 - 90 = 75$. (which is r of circle)

To find $\boxed{\text{period}}$ $= 32$; $\frac{2\pi}{32} = \frac{\pi}{16}$

Now you can write the formula.

* choose cos − because centre line is in the middle.

$y = (\text{amplitude})\cos(\text{period})(t - \underrightarrow{\text{phase shift}}) + \text{phase shift}\uparrow$

$\therefore\ y = 75\cos\frac{\pi}{16}(t-16) + 90$

See page 546

$\boxed{ex\,2}$ **tides** : (time, tide height)

period - from graph is 12 hours.

max point from graph (2, 7.8)

min point from graph (8, 1.4)

phase shift up

$\boxed{\text{Amplitude}} \Rightarrow \frac{7.8 + 1.4}{2} \doteq \boxed{4.6 \text{ centre line}}$

$\therefore \boxed{\text{amplitude}} = 7.8 - 4.6 \doteq 3.2$.

$\boxed{\text{Period}} = 12\text{h}$ so period $= \frac{2\pi}{12}$ or $\frac{\pi}{6}$

$\boxed{\text{Phase shift}}$ - from start to max point

$\doteq 2 \leftarrow$ phase shift right

* use cos because centre line is in middle.

$$y = 3.2 \cos \frac{\pi}{6}(x - 2) + 4.6.$$

TO use: what is the tide height at 16:15?

* put into decimal $16 \frac{15}{60} = 16.25$.

$$y = 3.2 \cos \frac{\pi}{6}(16.25 - 2) + 4.6$$

~ 5.8 m.