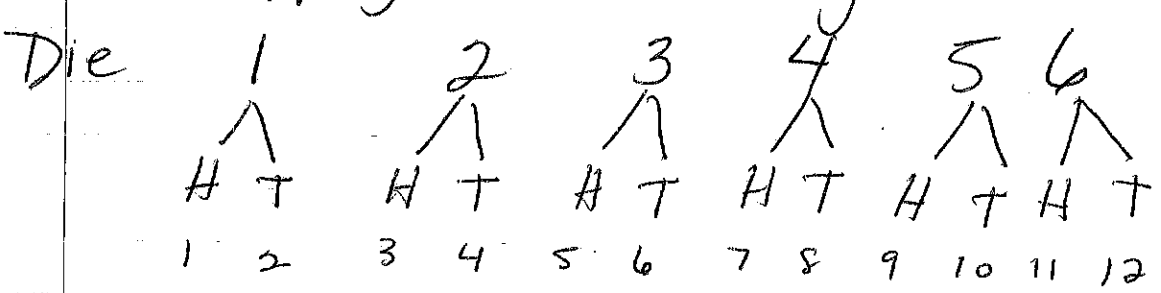
CH 8 - PC Math 12

8.1 The Fundamental Counting Principle

Draw a tree diagram to show all outcomes of flipping a coin & rolling a 6 sided die.



— 12 possibilities

Fundamental Counting Principle $n_1 \cdot n_2 \cdot n_3 \cdots n_\infty$

n_1 - ~~one~~ number of 1st object (sides on die = 6)

n_2 - number of 2nd object (sides on coin = 2)

$n_1 \cdot n_2$ = total # of outcomes

$6 \times 2 = 12$

ex 1 How many licence plates can be made with 3 letters than 3 numbers

letters in alphabet = 26

of numbers (1 digit) = 10 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

∴ # of possible plates = ~~26x10x260~~

$\underset{1st}{26} \cdot \underset{2nd}{26} \cdot \underset{3rd}{26} \cdot \underset{1st}{10} \cdot \underset{2nd}{10} \cdot \underset{3rd}{10} = 17576000$

8.1 * if choices cannot repeat
reduce by 1 each choice.

ex 2 ~~A lock can have 4 numbers~~

How many ways can you form a 3 letter word from the following letters:

A, B, C, D, E, F ← 6 letters w 3 slots

$6 \cdot 6 \cdot 6 = 216$

* NO REPITITION

$6 \cdot 5 \cdot 4 = 120$

* reduce by 1 * reduce again

8.2 Permutations of Different Objects

* the order matters!

Combination locks should be called Permutation locks!

How many 7 letter permutations can be made from the word KELOWNA

K L A _ _ _ _

7 · 6 · 5 · _ · _ · _ · _

① choose the first letter
* you cannot use this again so # of choices is reduced by 1

② once the second letter is chosen, it can't be used again so... the third letter is out of 5 remaining letters ... repeat ...

this can be written as

FACTORIAL NOTATION * 0! = 1

$$n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$$

for the above example:

there are $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ permulations

or $7! = 5040$

Permutation

$$_nP_r = \frac{n!}{(n-r)!} \quad n \geq r$$

n = total number of objects
r = how many taken at a time

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ex1 You have 78 songs on your IPOD. You want to listen to 5.

$$_{28}P_5 = \frac{28!}{(28-5)!} = \frac{28!}{23!} = 11793600 \text{ ways to choose 5 songs}$$

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* remember $0! = 1$ so if you choose 6 of 6 $\quad _6P_6 = \frac{6!}{(6-6)!} = \frac{720}{1}$

$= 720$

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* A visual to help w permulations:

You have 6 choices, total, you choose 3

$$_6P_3 = \frac{6!}{(6-3)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 6 \cdot 5 \cdot 4 = 120$$

8.3 Permutations + Identical Objects

$$\frac{n!}{r!}$$

$n!$ ← total objects

$r!$ ← identical objects

When a permutation has identical objects, the number decreases!

For example - Ryynanen

* if you were to make a permutation of my name - there are 8 letters but how could you tell if you used the 1st y or the second? the 3rd n or the 1st?

You can't → so you have to record that the words would be the same.

nRyaneyhRyanney* both make the same word.

1st 2nd

So for permutations with identical objects record each repeated letter

Ryynanen - 8 letters but 2y's and 3n's

$$\frac{8!}{2!3!} = 3360 \text{ ways}$$

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdots}$$

8.4 Combinations

$${}_nC_r = \frac{n!}{(n-r)!\,r!} \qquad n \geq r$$

* order does NOT matter

With combinations the results are viewed out of order
- choosing a blue, black then red ball is thought the same as a black, blue then red ball.

* Choosing the numbers for the lottery
... it doesn't matter if you pick 49 1st then 12 → just the result matters

Choose from 1-49; pick 7 numbers
How many combinations are possible?

$n=49 \quad r=7$

$${}_nC_r = \frac{49!}{(49-7)!\,7!} = \frac{49!}{42!\,7!} = \frac{49\cdot48\cdot47\cdot46\cdot45\cdot44\cdot43}{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

$$= 85\,900\,584$$

* Choose 3 people from a group of 10 to run prom

$${}_{10}C_3 = \frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!} = 120 \text{ ways}$$

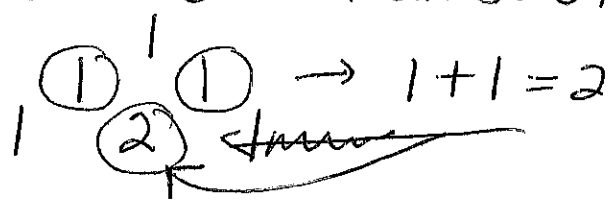
* to put on a calculator
$10! \div (7! \times 3!) =$

8.5 Pascals Triangle

```
          1          Row 1
        1   1        Row 2
      1   2   1      Row 3
    1   3   3   1
  1   4   6   4   1
  :   :   :   :   :
```

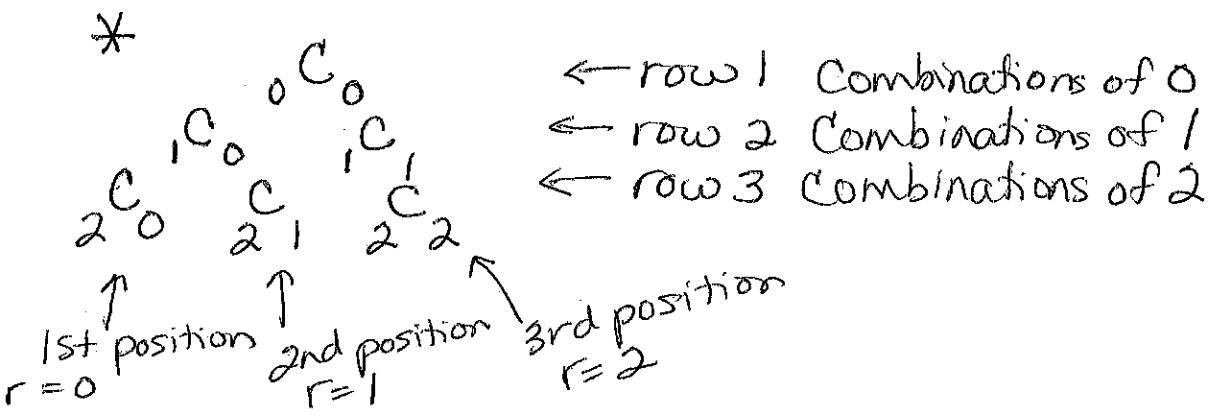
* note patterns: ① 1 is always 1st + last #

② # under 2 other numbers is sum



③ diagonal patterns
1, 2, 1, 3, 1, 4 ...
both from right + left.

④ Its a combination △.



To find $_{11}C_8$ in Pascal's triangle

+1 ↑ ↖ +1 ∴ 9th row 12th term

8.6 Binomial Theorem

Refer to Pascal's triangle to see pattern.

$(x+y)^3 = (x+y)(x+y)(x+y)$

$= (x^2+2xy+y^2)(x+y)$

$= x^3 + 3x^2y + 3xy^2 + y^3$

1 3 3 1 * this is the 4th row of pascal's Δ.

* now look at the exponents

$x^{\boxed{3}} + 3x^{\boxed{2}}y^{\Delta} + 3x^{\boxed{1}}y^{\Delta\Delta} + y^{\Delta\Delta\Delta}$

powers for x → 3, 2, 1, 0

powers for y → 0, 1, 2, 3

→ Using these patterns we can expand any binomial without all the steps

$(x+y)^5$ ⇒ use 6th row of pascal's Δ
1, 5, 10, 10, 5, 1

$1x + 5xy + 10xy + 10xy + 5xy + 1y$

* now add in exponents on x

$1x^5 + 5x^4y + 10x^3y + 10x^2y + 5x^1y + 1y$

then y → $1x^5 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1y^5$

* Now lets use different numbers & letters $(2a-3)^4$ ← 5th row * 1, 4, 6, 4, 1

* remember 5th row is ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

→ use $(x+y)^4$ then sub in

$x^4 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + y^4$

* now sub in $2a = x$ and $-3 = y$

$(2a)^4 + 4(2a)^3(-3)^1 + 6(2a)^2(-3)^2 + 4(2a)(-3)^3 + (-3)^4$

$= 16a^4 - 96a^3 + 216a^2 - 216a + 81$

Binomial Theorem (Combinations)

* $(x+y)^n = {}_nC_0 x^n + {}_nC_1 x^{n-1}y + {}_nC_2 x^{n-2}y^2 \ldots$

Binomial Theorem (using algebra)

$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 \ldots$

General Binomial Theorem (* to find a specific term)

${}_nC_{k-1}\, x^{n-(k-1)}y^{k-1}$ ← to find kth term of $(x+y)^n$

ex 7th term of $(x-3)^9$

${}_9C_{(7-1)}\, x^{9-(7-1)}y^{7-1} = {}_9C_6 x^3 y^6$ sub in → ${}_9C_6 (x)^3(-3)^6$

$= 84x^3(729) = 61236x^3$