Polynomials Pre-Pretest

Name: Key.

Make a reference card/sheet with the following terms, definitions and theorems: (you may use this during the test)

- Definition of a polynomial
- Polynomial terms (example: degree)
- Odd and even functions
- Remainder theorem
- Factor theorem
- Multiplicity
- Integer zeroes theorem

Polynomials Pretest

Section 1 - Identifying Polynomials (/20%)

Are the following equations polynomials? If it is, give the degree. If not, give one property of a polynomial that it does not satisfy.

$g(x) = x^2 - 2x + 1$

Yes, degree 2

$f(x) = (x-1)^{10}$

yes, degree 10

$h(x) = \frac{(x+1)^2}{(x+1)}$

no

$x \neq -1$

$k(x) = \frac{x^3-3x^2+x-3}{x^2+1}$

this means divides

yes. $(x^2+1 \,|\, x^3-3x^2+x-3)$

degree $= 3-2 = 1$

$q(x) = \sqrt{x^2}$

no, not smooth

(same as $\rightarrow q(x) = |x|$)

$r(x) = -5$

yes

degree 0

$p(x) = \frac{(x-2)^2}{x^4+1}$

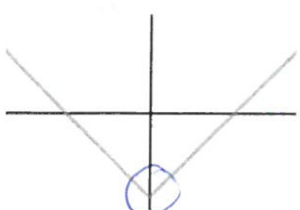
no

degree -2 not allowed

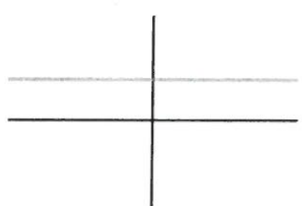
$m(x) = 0$

yes.

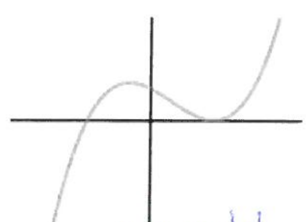
no degree



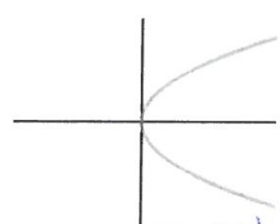
no, not smooth



yes, even

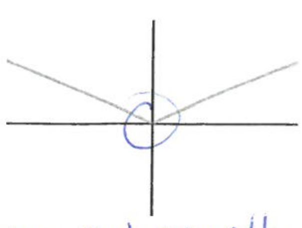


yes, odd.

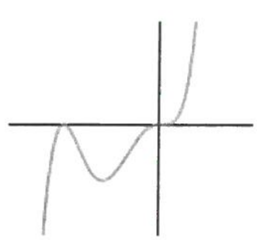


no, not all real for x.

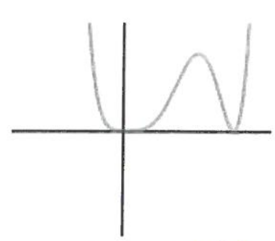
ie $x \neq -1$



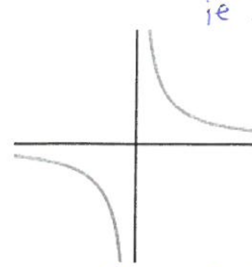
no, not smooth



yes. odd



yes, even

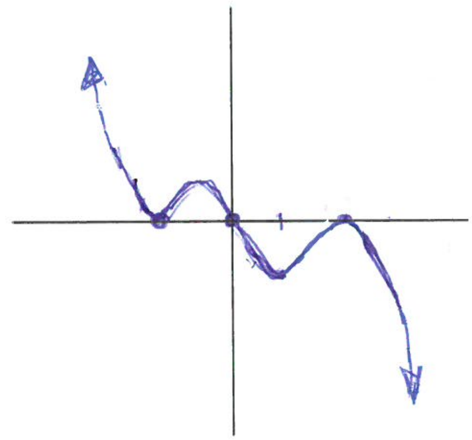


no, $x \neq 0$

Section 2 - Graphing (/20%)

Find the leading term, y-intercept, zeros and multiplicities of each zero, then graph it.

$$f(x) = -2x(x-2)^2(x+1)^2$$



Zeros: $x=0$ $m_0 = 1$

$x = 2$ $m_2 = 2$

$x = -1$ $m_{-1} = 2$.

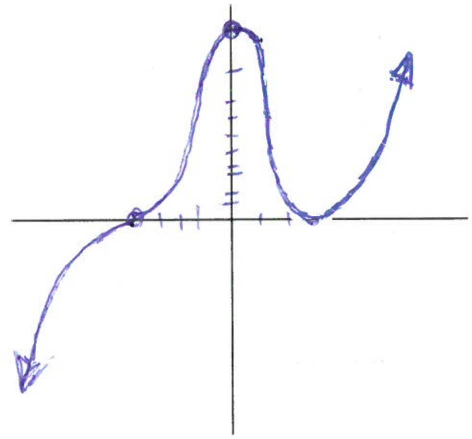
y-intercept: $f(0) = 0$

end behavior. → leading term.

$-2x^5$ negative and odd degree.

Find the leading term, y-intercept, zeros and multiplicities of each zero, then graph it.

$$f(x) = \frac{1}{25}(x+4)^3(x-2)^2$$



zeros: $x=2$ $m_2 = 2$

$x=-4$ $m_{-4} = 3$

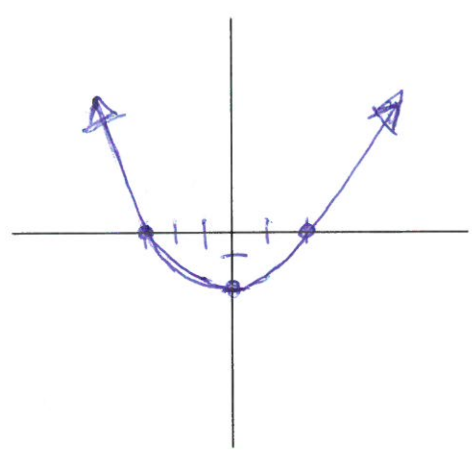
y-intercept: $f(0) = \frac{64 \times 4}{25} = \frac{256}{25} = 10\frac{6}{25}$

~~lead~~ leading term → $\frac{1}{25}x^5$

positive, odd degree.

Find the leading term, y-intercept, zeros and multiplicities of each zero, then graph it.

$$f(x) = \frac{1}{30}(x+3)^3(x-2)$$



zeros: $x=-3$ $m_{-3} = 3$

$x = 2$ $m_2 = 1$

y-intercept: -1.8

leading term: $\frac{1}{30}x^4$

positive & even degree

Section 3 - Understanding Polynomials (/30%)

Give the leading term, y-intercept and the end behavior of each of the following polynomials:

$x^3 - 21x^2 + 147x - 343$

Leading term x^3

Y-intercept -343

End behavior:

$2(3x-1)^2(x+3)^2$

Leading coeff : $2 \times 3^2 \times 1 = 18$
Degree : $2 \times 2 = 4$ $\Big\}$ Term $18x^4$

y-int: $2(2)^2(3)^2 = 72$

End behavior:

$-x(x-1)(x-2)(x-3)$

Leading coeff: -1
Degree : 4 $\Big\}$ $-x^4$

Y-intercept : 0

End behavior:

$(2-x)^2(3-x)^3$

Leading coeff: $(-1)^2(-1)^3 = (-1)^5 = -1$
Degree : 5 $\Big\}$ $-x^5$

Y-intercept : $(2)^2(3)^3 = 108$

End behavior:

What value of m will ensure that $x+3$ is a factor of $mx^3 - 2x^2 + x - 6$?

If it is a factor, by remainder theorem, $f(-3) = 0$

$\therefore$ $f(-3) = m(-3)^3 - 2(-3)^2 + (-3) - 6 = 0$

$\Rightarrow$ $-27m - 18 - 3 - 6 = 0$ and $m = -1$

$\Rightarrow$ $-27m = +27$

For each polynomial, determine one factor of the form $x - a$ where a is an integer:

$x^4 + 3x^3 - 9x^2 - 23x - 12$

$a \in \{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$

$f(1) = -40$

$f(-1) = 0 \Rightarrow (x-(-1))$ is a factor.

$x^3 + x^2 - 8x - 12$

$a \in \{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$

$f(1) = -18$ $f(2) = -16$

$f(-1) = -4$ $f(-2) = 0. \Rightarrow (x-(-2))$ is a factor.

$x^3 - 21x^2 + 147x - 343$

$343 = 7^3$

$a \in \{\pm1, \pm7, \pm49, \pm343\}$

$f(1) = -216$

$f(-1) = -512$

$f(7) = 0. \Rightarrow (x-7)$ is a factor.

$x^5 - 17x^4 + 78x^3 - 142x^2 + 113x - 33$

$a \in \{\pm1, \pm3, \pm11, \pm33\}$

$f(11) = 0 \Rightarrow (x-11)$ is a factor.

Section 4 - Factoring Polynomials (/30%)

Factor the following polynomials:

$f(x) = x^4 - 9x^2 + 4x + 12$

First find factors: test: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$f(1)=8$ $f(-1)=0$ $f(2)=0$ $\Rightarrow (x+1)(x-2)$ factors

Divide by

$$x^3 - x^2 - 8x + 12$$
$$(x+1)\overline{)x^4 + 0x^3 - 9x^2 + 4x + 12}$$
$$-(x^4 + x^3)$$
$$-x^3 - 9x^2$$
$$-(-x^3 - x^2)$$
$$-8x^2 + 4x$$
$$-(-8x^2 - 8x)$$
$$12x + 12$$
$$12x + 12$$
$$0$$

$$x^2 + x - 6$$
$$x-2\overline{)x^3 - x^2 - 8x + 12}$$
$$x^3 - 2x^2$$
$$x^2 - 8x$$
$$-(x^2 - 2x)$$
$$-6x + 12$$
$$-6x + 12$$
$$0$$

$(x+1)(x+2)(x^2+x-6)$

$(x+1)(x+2)(x+3)(x-2)$

$f(x) = x^3 - 13x^2 + 56x - 80$

Find factors: test: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 10 \ldots$

$f(1)=-36$ $f(-1)=-150$ $f(2)=-12$ $f(-2)=-252$

$f(4)=0$ so $(x-4)$ is a factor

$$x^2 - 9x + 20$$
$$x-4\overline{)x^3 - 13x^2 + 56x - 80}$$
$$-(x^3 - 4x^2)$$
$$-9x^2 + 56x$$
$$-(-9x^2 + 36x)$$
$$20x - 80$$
$$20x - 80$$
$$0$$

$(x-4)(x^2-9x+20)$

$(x-4)(x-4)(x-5)$

$f(x) = x^4 - 9x^3 + 13x^2 + 9x - 14$

$f(1)=0$ two factors $(x-1)$ and $(x+1)$

$f(-1)=0$

Just to try something different

Divide by $(x-1)(x+1) = x^2 - 1$ (saves a step)

$$x^2 - 9x + 14$$
$$x^2 + 0x - 1\overline{)x^4 - 9x^3 + 13x^2 + 9x - 14}$$
$$-(x^4 + 0x^3 - x^2)$$
$$-9x^3 + 14x^2 + 9x$$
$$-(-9x^3 + 0x^2 + 9x)$$
$$14x^2 + 0x - 14$$
$$14x^2 + 0x - 14$$
$$0$$

So we have.

$(x-1)(x+1)(x^2-9x+14)$

$(x-1)(x+1)(x-2)(x-7)$

$f(x) = x^3 + 15x^2 - 33x - 847$ $847 = 11 \times 11 \times 7.$

$f(7)=0 \Rightarrow x-7$ is a factor.

$$x^2 + 22x + 121$$
$$x-7\overline{)x^3 + 15x^2 - 33x - 847}$$
$$-(x^3 - 7x^2)$$
$$22x^2 - 33x$$
$$-(22x^2 - 154x)$$
$$121x - 847$$
$$-(121x - 847)$$
$$0$$

$(x-7)(x^2+22x+121)$

$(x-7)(x+11)^2$